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SH Waves Diffusion in Fluid-Saturated Medium

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Abstract: Problem statement: Characteristics of the seismic waves have been studied mainly the amplitudes, attenuation, frequency, porosity and the velocity of the waves. In this study, the SH-waves were studied for the diffusion which was related to the diffraction of SH-waves in the saturated medium. **Approach:** The elastic wave equation was being used to show the SH-waves. A comparison was made of the propagating SH-waves in the fluid-saturated mediums that were categorized into two distinct groups: Insoluble and soluble mediums. **Results:** The discussions on fluid density in the mediums showed that high density fluid promotes diffusive characteristic while low density fluid endorses non diffusive SH-wave. However, the diffraction had induced significant shock wave in the medium saturated with low density fluid. **Conclusion:** These results indicated that the linear seismic waves are able to transform into nonlinear waves.

Key words: Diffusion, attenuation, shock waves, diffraction

INTRODUCTION

Seismic waves are well-known for being the plane waves. P-, SV and SH-waves are the seismic plane waves that are generated during the quake. In extreme cases, the researchers consider the use of S-wave in detecting the surface crack in locomotive's axles of high speed trains or steam-generator pipes in nuclear power plants (Claudio, 2005) and for piezoelectric material. Propagation behavior of SH-wave is studied for initial stress (Qian *et al*., 2009). Later, the dissipation of SH-wave in piezoelectric media was also studied (Du *et al*., 2009). Biot's theory is implied for studying SH-waves especially the fluid saturated porous media (Ghorai *et al*., 2009).

The dependence of P- and S-wave attenuation to strain amplitude and frequency in saturated medium is done (Mashinskii, 2009). Investigation on saturation on horizontal and vertical of porous soils was carried out (Yang and Tadanobu, 2000). As the wave's amplitude increases, the frequency increases too. Effects of saturation on the velocities of P- and S-waves in several type of soil are studied and the outcomes show that the velocity is higher in dense sand. Similar outcome is found (Kahraman, 2007). The thickness of the porous medium is related to the frequency of the waves. The frequency decreases when the thickness of the medium increases (Wang *et al*., 2009).

Hence, this study aims to probe the connection of SH-waves refraction velocity and the attenuation. The derivation of SH-waves will be shown follow by the analytic discussion. Some figures will be plotted to give better viewing pleasure.

MATERIALS AND METHODS

Problem formulation: The governing equations:

$$\mu\nabla^2 u + (\lambda+\mu)\nabla(\nabla u) + \rho F = \rho\frac{\partial^2 u}{\partial t^2} \qquad (1)$$

Boundary conditions for z = 0:

$$\begin{aligned}\sigma_z &\equiv (\lambda+\mu)\frac{\partial u_z}{\partial z} + \lambda\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) = 0\\ \tau_{xz} &\equiv \mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = 0\\ \tau_{yz} &\equiv \mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) = 0\end{aligned} \qquad (2)$$

Initial conditions for t = 0:

$$u(x,y,z) = 0 \qquad (3)$$

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$$\frac{\partial u_x}{\partial t} = \frac{\partial u_y}{\partial t} = \frac{\partial u_z}{\partial t} = 0 \quad (4)$$

In linear elasticity for isotropic medium, λ and μ denote the Lame parameters for the stress σ_z, τ_{xz}, τ_{yz} and the displacements u_x, u_y and u_z are continuous everywhere. F is the body force in the direction of x, y, z respectively and ρ is the density.

Provided that ω is the angular velocity or frequency, k is the wave number, c is the wave velocity along with the dispersion relation ω = ck, the group velocity of a wave is the velocity with the overall shape of the wave's amplitudes which is also known as envelope of the waves. In other words, the phase velocity is the average velocity of the components, given by $V_P = \omega/k$. The group velocity is velocity of the envelope, given by $V_g = d\omega/dk$ (Bhatnagar, 1979).

When this is applied to the problem of SH-wave's propagation in fluid saturated medium, the group velocity reflects the apparent velocity of the surface displacement or the overall shape of the SH-wave's amplitude at the boundary of the medium. The envelope is formed by the phase velocity of SH-waves. In this research, the apparent velocity V_{app} or surface displacement velocity is measured along the boundary of the similar density medium in accordance to Snell law:

$$\frac{V_{app}}{\sin f} = \frac{c}{\sin e} \quad (5)$$

e = The incident angle
f = The refraction angle made by the P- and S-waves

However, this research aims to study S-waves in horizontal direction only. For the case of fluid-saturated medium, there exists variation in density within medium (Wang *et al*., 2009; Sharma, 2004). There exists slowness for fluid-saturated medium (Keith and Crampin, 1977; Sharma, 2007) i.e., the apparent velocity of the displacement at the boundary is slower than phase velocity of the wave in the medium that reads:

$$V_{app} < c \quad (6)$$

In this study, the elastic wave equation will be solved for extracting SH-wave's displacements in the different types of medium that satisfied (5) and (6). By using the divergence operator:

$$V^2 u = \nabla(\nabla u) - \nabla \times (\nabla \times u) \quad (7)$$

Equation 1 is reduced to:

$$\alpha^2 \nabla(\nabla \cdot u) - \beta^2 \nabla \times \nabla \times u + F = \frac{\partial^2 u}{\partial t^2} \quad (8)$$

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \beta = \sqrt{\frac{\mu}{\rho}} \quad (9)$$

α and β are the velocities for the P-wave and S-wave and Eq. 8 is the elastic wave equation (Pujol, 2003). Hence, this modeling only valid for elastic medium and it is necessary to reduce the Right Hand Side (RHS) of (8) by letting:

$$u = \exp[ik(x - ct)] \quad (10)$$

By inserting second order derivative of (10) into RHS of (8), the equation reads:

$$\alpha^2 \nabla(\nabla \cdot U) - \beta^2 \nabla \times \nabla \times u + F = -\omega^2 u \quad (11)$$

Since elastic wave consists of irrotational P-wave and solenoidal by S-wave (Pujol, 2003), the displacement shall be written as:

$$u = u_p + u_s \quad (12)$$

For irrotational P-waves, the vorticity $\nabla \times u_p = 0$. Thus, the relation (12) reads:

$$\nabla u_p \neq 0, \nabla \times u_p = 0, \nabla u_s = 0, \nabla \times u_s \neq 0 \quad (13)$$

By inserting (12) and (13) into (11), the equation yields:

$$\alpha^2(\nabla^2 u_p + k_p{}^2 u_p) + \beta^2(\nabla^2 u_s + k_s{}^2 u_s) \quad (14)$$

$$\omega = \frac{\alpha}{k_p}, \frac{\beta}{k_s}, F = 0 \quad (15)$$

In the following, the Eq. 16 is the Helmholtz equations (Gang *et al*., 2004) for P- and S-waves:

$$\nabla^2 u_p + k_p{}^2 u_p = 0, \nabla^2 u_s + k_s{}^2 u_s = 0 \quad (16)$$

Here, the Helmholtz equations are solved by utilizing Hansen vector (Pujol, 2003) that gives:

$$u_p = A(la_x + na_z)\exp\left[i\omega\left(t - \frac{lx + nz}{\alpha}\right)\right] \quad (17)$$

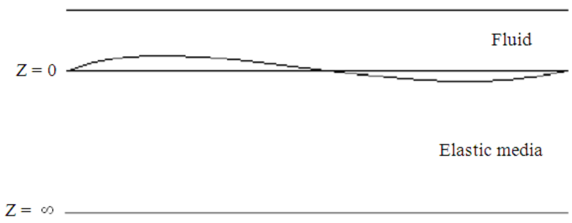

Fig. 1: The medium with depth z

$$u_{SV} = B(-na_x + la_z)\exp\left[i\omega\left(t - \frac{lx+nz}{\beta}\right)\right] \quad (18)$$

$$u_{SH} = Ca_y \exp\left[i\omega\left(t - \frac{lx+nz}{\beta}\right)\right] \quad (19)$$

a_x, a_y and a_z are the unit vectors while l and n are the vector components. Only Eq. 19 will be considered in this study since the aim is to study the SH-wave only. SH-wave vanishes when the depth goes infinity as illustrated in Fig. 1. Thus, the amended Eq. 19 or SH-wave on which the amplitude reduces with depth is:

$$u_{SH} = Ca_y \exp\left[ik\left(ct - x - \eta_\beta z\right)\right], \eta_\beta = \sqrt{\frac{c^2}{\beta^2} - 1} \quad (20)$$

η_β is always positive. The velocity of the SH-wave, β is measured at the boundary of the medium or z = 0 and it is similar to the apparent velocity V_{app} of the surface displacement.

For the displacement normal to the propagation direction by means of quantity η_β and amplitude C, the Eq. 20 for y-direction of displacements with amplitude reduces with depth z gives:

$$u_{SH} = Ca_y \exp\left[-ik\eta_\beta z\right]\exp\left[ik(ct - x)\right]\omega_1 = \eta_\beta k \quad (21)$$

Here, quantity η_β refers to the refracted wave velocities that give $\omega_1 = \eta_\beta k$ while the vector a_y shows the displacement is at y-direction while the SH-wave propagates in x-direction. From wave terminology, the term $\exp\left[-ik\eta_\beta z\right]\exp\left[ik(ct - x)\right]$ in Eq. 21 will show the plane waves that are diffusive with depth z of $\omega_1 = \eta_\beta k$ is in complex (Bhatnagar, 1979). The diffusive waves are associated with attenuation of the amplitudes with the time due to certain dissipation mechanisms present in the system.

Equation 21 should be polarized to give the real quantities (frequency) equation that reads (Pujol, 2003):

$$u_{SH} = \left[C\exp\left(-ik\eta_\beta z\right)\right]\exp\left[ik\left(ct - x\right)\right] \quad (22)$$

Next, the roles of η_β will be shown. For the fluid-saturated medium, there comes the slowness induced by refraction (Keith and Crampin, 1977; Sharma, 2007). The envelope velocity at medium surface is different with wave velocity in the medium after the slowness or $V_{app} \neq c$. For particular case, the SH-wave velocity in the medium is greater than the envelope's velocity that yields:

$$c > \beta, \ \beta = V_{app} \quad (23)$$

Here, we propose the relation for another two types of medium conditions such that:

$$c = \beta, \ \beta = V_{app} \quad (24)$$

$$c < \beta, \ \beta = V_{app} \quad (25)$$

Here, we mark that the relations (23) and (25) are meant for the insoluble medium such that the variation of velocities c and β is significant. When the velocity c is similar to β, we presume this will explain the soluble medium such that the fluid mixes well with the medium to give similar velocity.

Yang and Tadanobu (2000) and Kahraman (2007) show that the high density medium promotes high wave's velocity. Hence, the relation (23) will only be present when the low density fluid is saturated in the insoluble medium; the low density fluid will reshuffle the ray velocity or reduce the SH-wave velocity. Eventually, the relation (25) is meant for the high density fluid saturated in the insoluble medium.

The detailed explanations about the tie between medium's solubility and the fluid's density will be discussed next for showing the vital roles play by the relations (23-25) especially the attenuation's characteristic. When condition (25) is applied to the quantity η_β in Eq. 20, a complex solution will be obtained for η_β that reads:

$$\eta_\beta = i\eta_\beta \quad (26)$$

This indicates an amendment is required for Eq. 22 to give (27). Hence, the SH-wave's displacements for 3 types of mediums are:

$$u_{SH} = \left[C \cdot \exp\left(k\eta_\beta z\right)\right]\exp\left[ik\left(ct - x\right)\right] \text{for } c < \beta \quad (27)$$

$$u_{SH} = \left[C \cdot \exp\left(ik\eta_\beta z\right)\right]\exp\left[ik\left(ct - x\right)\right] \text{for } c = \beta \quad (28)$$

$$u_{SH} = \left[C \cdot \exp\left(ik\eta_\beta z\right)\right]\exp\left[ik\left(ct - x\right)\right] \text{for } c > \beta \quad (29)$$

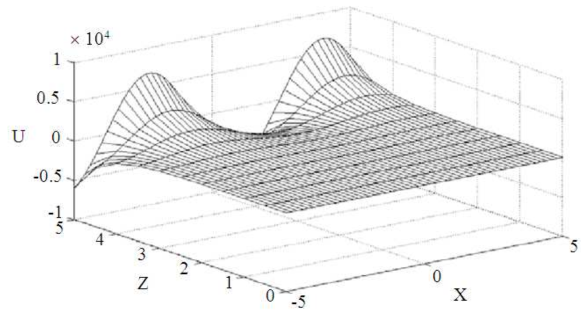

Fig. 2: The diffusive characteristic by SH-wave's propagation in the medium saturated with high density fluid. $c< \beta$, $-5\leq x\leq 5$, $k = 1$, $\beta = 4$, $c = 2$

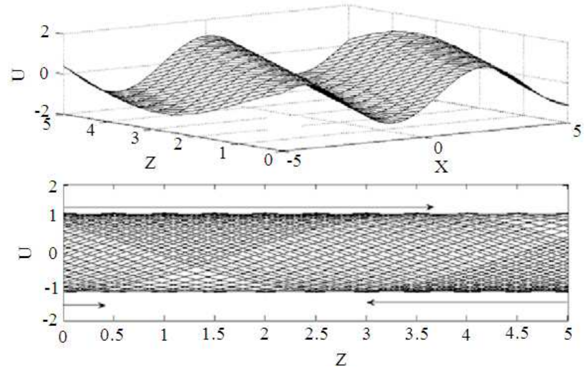

Fig. 3: The non-diffusive characteristic by SH-wave's propagation in the medium saturated with low density fluid. $c> \beta$, $-5\leq x\leq 5$, $k = 1$, $\beta = 2$, $c = 4$

Equation 27-29 show that the SH-wave's propagation in x-direction whiles the diffusion is in z-direction. The Eq. 27-29 are arranged such that the density of the saturated fluid reduces from $c<\beta$-$c>\beta$. Subsequently, Fig, 2 and 3 are plotted respectively.

RESULTS

Figure 2 is plotted for the diffusive SH-waves with $c<\beta$, $-5\leq x\leq 5$, $k = 1$, $\beta = 2$, $c = 4$. High density fluid promotes the diffusive characteristic for propagating SH-waves in fluid saturated medium. The amplitude diminishes at medium surface or $z = 0$.

Insoluble medium with low density fluid (29) is non-diffusive given that the term $\omega_1 = \eta_\beta k$ is in real (Bhatnagar, 1979).

Figure 3 is plotted for the non-diffusive SH-waves with $c > \beta$, $5\leq x\leq 5$, $k = 1$, $\beta = 2$, $c = 4$. However it is shown in Fig. 3 the compression exists when the SH-waves are polarized in x-y direction. The compression is related to shock wave which is formed by the wave diffraction (Han and Yin, 1993).

DISCUSSION

For soluble medium with $c = \beta$, Eq. 28 gives:

$$u_{SH} = C\exp\left[ik\left(ct - x\right)\right], \eta_\beta = 0 \quad (30)$$

Relation (30) implies that the SH-wave in soluble medium is non-diffusive plane waves and $\eta_\beta = 0$ explains that there is no diffraction to be done. Insoluble medium with displacement (27) is diffusive given that the term $\omega_1 = \eta_\beta k$ is in complex in according to (26).

CONCLUSION

The studies show that the SH-waves in soluble medium are non-diffusive plane waves. However, the discussion on fluid density has linked the SH-waves to the diffusion. High density fluid in insoluble medium promotes diffusive SH-waves while low density fluid in insoluble medium promotes non-diffusive SH-waves. The non-diffusive waves have induced shock wave in the insoluble medium after diffraction.

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