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General Linear Models in a Missing Outcome Environment of Clinical Trials Incorporating with Splines for Time-Invariant Continuous Adjustment

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Abstract: Missing data is a common occurrence in longitudinal studies of health care research. Although many studies have shown the potential usefulness of current missing analyses, e.g., (1) Complete Case (CC) analysis; (2) imputation methods such as Last Observation Carried Forward (LOCF), multiple imputations, Expectation-Maximization algorithm approach; and (3) methods using all available data such as linear mixed model and generalized estimation equations approach, the CC analysis or LOCF imputation method have been popular due to their simplicity of execution regardless of some critical drawbacks. The proposed approach employs the generalized least squares method using all available data without deletion or imputations for missing outcomes, producing the best linear unbiased estimate. A simulation study was conducted to compare the proposed approach to commonly used missing analyses under each missing data mechanism and showed the validity of the proposed approach, especially with the first order autoregressive correlation structure. B-spline is applied to the proposed model to manage non-linear relationships between outcome and continuous covariate. Application to a cell therapy clinical trial is presented.

Keywords: Missing Outcome, Clinical Trials, General Linear Models, B-Spline

Introduction

Missing data plagues health care research, weakening a study's conclusion. A review of randomized controlled trials reported 89% of 71 trials had partially missing primary outcomes and 18% of trials had more than 20% missing outcomes (Wood *et al*., 2004). Although missing data occur for a variety of understandable reasons (e.g., subject non-response, subject dropout, technical device error, or data entry error), the absence of information undermines the research effort (Schafer and Graham, 2002). The missing data problems become more acute in longitudinal studies where the commitment of subjects to the study over a period of time can lag. Hence, many helpful approaches handling missing data have been introduced for decades; in turn, this has resulted in new methods for managing missing data.

Missing data analyses have been remarkably extended since the missing data mechanism was defined by Little and Rubin (1987): Missing Completely At Random (MCAR) when a missing value depends on neither observed nor unobserved values, Missing At Random (MAR) when a missing value depends only on the observed values, Missing Not At Random (MNAR) when a missing value depends on unobserved data. Many advanced imputation methods have been developed based on the missing data mechanism. Multiple Imputations (MI) introduced by Rubin (1978), is a distribution-based method sampling values from a distribution of observed data. Hence, it is valid only under MAR. The Expectation-Maximization (EM) algorithm approach is one of the likelihood-based methods (Rubin, 1976) finding maximum likelihood estimates for fractional data, also valid under MAR (Dempster *et al*., 1977). The mixed effect models and the Generalized Estimating Equations (GEE) are commonly used for repeated measurements in longitudinal study. Both can operate in unbalanced designs, hence, they incorporate all available data without deletion or imputation. The former one is valid when missing data is ignorable (i.e., MCAR or MAR) and the latter one is valid only when MCAR holds (Liang and Zeger, 1986). The pattern mixture (Little, 1993; 1995) and shared-parameter models (Wu and Bailey, 1988; 1989) have been developed for use in MNAR situation which is more complex since it requires the specification of missing process (Verbeke *et al.*, 2001; Michiels *et al.*, 2002; Hogan and Laird, 1997).

Yet in the face of superlative efforts to identify proper methods for managing missing data, utilization remains limited in practice. Among studies having missing primary outcomes (Wood *et al.*, 2004), 65% of trials carried out the Complete Case (CC) analysis for primary outcome and 24% of trials used the following imputation methods in order of frequency: Last Observation Carried Forward (LOCF), worst case value, regression-based imputation and MI. In addition, 8% of trials used repeated measures analyses such as the GEE approach (Wood *et al.*, 2004). Others also noted that most studies applied the CC analysis or LOCF imputation method for primary analyses (Fielding *et al.*, 2008). In fact, a major reason why the CC analysis and simple imputations such as LOCF imputation are the most commonly used procedures to manage missing data is the complexity of assumptions underlying the more advanced models (Little and Rubin, 1987). For researchers who are not specialists in missing data analyses, the CC analysis and simple imputations are relatively easy to implement. However, these simple approaches are valid only under MCAR which is less congenial than MAR in clinical trials. Besides, dropouts which are commonly occurred in clinical trials are usually regarded as MAR. Hence, simple approaches cannot avoid a serious bias unless the condition is met. Therefore, it is critical to find an approach which is as simple and straightforward as CC analysis or simple imputations such as LOCF, yet incorporating all available data which avoids additional assumptions by imputing non-existing data such as MI or the EM algorithm approach and also being valid under MAR.

The purpose of this study is to introduce an alternative method to manage incomplete outcome data, yielding the Best Linear Unbiased Estimate (BLUE) by employing the Generalized Least Squares (GLS) method to rearranged matrices in the model after considering missing outcomes without imputation or deletion of cases. In addition, B-spline is employed for managing non-linear relationships between outcome and continuous covariate which can be a potential confounder.

Materials and Methods

Proposed Approach

The proposed approach employed the GLS method, yielding the BLUE in general linear models (Seber and Lee, 2003). For example, suppose there is a trial evaluating blood pressure (Y) changes over time using two longitudinal measures at baseline ($X = 0$) and endpoint ($X = 1$) with n subjects, i.e., a design similar to the paired t-test. Assuming a constant variance σ^2 in both groups, this model can be specified as $E(Y) = \beta_0+\beta_1 X$ and $Var\ (Y) = \sigma^2 V$, where V is a correlation matrix allowing correlated outcomes (Diggle *et al.*, 2002). In the proposed approach, repeated time was included in the model as categorical variable and time-invariant covariates, e.g., gender, race, are considered if needed.

In the presence of missing data, we removed rows from the design matrix, both rows and columns from covariance matrix corresponding to the missing elements of y_{ij} in the outcome vector, where y_{ij} is the outcome value of the j^{th} time point in the i^{th} subject. Assuming that there are l missing values at baseline and m missing values at endpoint, the dimension of outcome vector becomes ($2n$-l-m) and the design matrix and correlation matrix are turned to be ($2n$-l-m)×2 and ($2n$-l-m)×($2n$-l-m), respectively, after removing the components corresponding to the missing outcomes and collapsing the subsequent structure. The GLS method can be applied to the collapsed model yielding BLUE, since the matrices remain full rank. The GLS estimates with missing data are formulated as:

$$\underline{\hat{\beta}}_c = (X_c{'}V_c^{-1}X_c)^{-1}X_c{'}V_c^{-1}\underline{y}_c$$

And its variance:

$$Var(\underline{\hat{\beta}}_c) = (X_c{'}V_c^{-1}X_c)^{-1}\hat{\sigma}_c^2$$

where, X_c and V_c are the collapsed design matrix and correlation matrix respectively, assuming that V_c is known. A constant variance can be estimated by $\hat{\sigma}_c^2 = (Y - X\underline{\hat{\beta}}_c)'V_c^{-1}(Y - X\underline{\hat{\beta}}_c)/(N-p)$, where N is the number of total observations and p is the number of parameters. In the simulation study, both exchangeable (or compound symmetry: Corr (Y_{ij},Y_{ik}) = ρ, $j \neq k$) and the first order Auto-Regressive (AR(1): Corr ($Y_{ij},Y_{i,j+k}$) = ρ_k, k = 1,… t-1, where t is the total number of repeated time) are considered as correlation structures. The GLS estimate of ρ is estimated by maximizing the Restricted Maximum Likelihood (REML) as specified below:

$$L(\rho; y_1,\ldots,y_A) = c - \frac{1}{2}\sum_{i=1}^{A}\log|V_i| - \frac{1}{2}$$
$$\sum_{i=1}^{A} r_i V_i^{-1} r_i - \frac{1}{2}\sum_{i=1}^{A}\log\left|X'_i V_i^{-1} X_i\right|$$

where, $r_i = y_i - X_i\left(\sum_{i=1}^{A} X'_i V_i^{-1} X_i\right)^{-1}\left(\sum_{i=1}^{A} X'_i V_i^{-1} y_i\right)$, c is an appropriate constant, A is the number of all paired data. Once the estimated correlation matrix $\hat{V}_c$ is obtained, it is used to solve for the parameter estimates.

B-Spline

The linearity of the parameters (-/+) is one of the underlying assumptions in general linear models when a continuous covariate is included in the model. In circumstances where the model includes a continuous covariate having non-linear relationships with outcomes, adding polynomial terms, e.g., x^2, x^3,… x^p is a commonly used method for explaining a nonlinear shape. As an alternative, we addressed the B-spline which is composed of polynomial non-negative pieces divided by knots, where a knot is a joint connecting two consecutive curves (de Boor, 2001). Let the knots be non-decreasing real numbers as, $t_0 \le t_1 \le \cdots \le t_k \le t_{k+1}$, where t_1, …, t_k are k number of interior knots and t_0, t_{k+1} be two endpoints and let us define the step function as:

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & \textit{other wise} \end{cases}$$

Then the polynomial B-spline can be formulated as:

$$B_j^n(t) = \frac{t - t_j}{t_{j+n} - t_j} B_j^{n-1}(t) + \frac{t_{j+n+1} - t}{t_{j+n+1} - t_{j+1}} B_{j+1}^{n-1}(t)$$

where, $B_j^0(t) = u(t - t_j)u(t_{j+1} - t)$, j = 1, …, k+n+1 and n is the degree of the B-spline. In this study, the parametric B-spline with 3 degrees, called cubic B-spline, $B(x) = \sum_{j=0}^{k+4} \beta_j B_j^3(t)$, was applied in the proposed approach, generating a cubic curve for each segment between knots.

Comparison of Missing Analyses

Results from the above approach were compared to several popularly used alternative models.

The naïve way of managing missing data is the CC analysis which discards all incomplete cases. It was applied under MCAR only. The LOCF imputation method was also applied under MCAR only, as one of simple imputation methods by replacing missing values with the latest previous observation.

Among advanced imputation methods, MI and the EM algorithm approaches were applied under all missing data mechanisms. The MI method replaces missing values with an inference of n datasets drawn from the conditional distribution of missing data. In the simulation study, the averaged values of five datasets were imputed using R function mice (van Buuren and Groothuis-Oudshoorn, 2011). In the EM algorithm approach, the missing elements were drawn from the multivariate normal model with estimates obtained by the maximum likelihood estimation method using the EM algorithm, incorporating a general iterative algorithm by R package NORM (Dempster *et al*., 1977).

The LMM and GEE approach were applied under all missing mechanisms. In the LMM, the model included the random intercept for explaining subject-specific variability and applied in R function lme. For the correlation estimate, the gold standard was assigned to an initial value. The GEE was also applied in R function geeglm (Højsgaard *et al*., 2005) as a standard longitudinal data analysis incorporating all available data without imputation.

The random-effect pattern mixture model was applied only under MNAR situation (Hedeker and Gibbons, 1997). Defining missing patterns, we generated a dummy variable (D) for each pattern and included it in the model. In the example of trial with two time points, there can be two possible patterns as presented below:

Pattern	Time1	Time2	D
1	O	X	0
2	X	O	1

Data

A simulation study was conducted in R version 2.15.2. to assess the validity of the proposed approach by comparison to commonly used missing analyses (CC analysis, LOCF, LMM, MI, the EM algorithm approach, the GEE approach) under each missing data mechanism with various missing rates in possible scenarios based on the number of parameters. The comparison between models was based on bias, Standard Deviation (Std.Dev) of estimates, mean Standard Error (SE) and 95% Coverage Probability (CP) which is the proportion of the time that the 95% confidence interval contains the true value of interest. For convenience, the following condition was employed for the sampling:

$$Y \sim MVN\left(X\beta, \sigma^2 V\right)$$

where, σ^2 is a constant variance and V is a positive definite correlation matrix. One thousand individuals were randomly generated in a dataset and one thousand datasets were generated for each scenario. Since it is a longitudinal setting, the total number of outcomes were 1000× (time points), e.g., two thousand outcomes for a model including two time points.

Missing data were generated based on each missing data mechanism. Under MCAR, it was randomly generated from the given missing rates. For MAR or MNAR, it was generated based on the criteria specified upper quartile of missing rate from the normal distribution with gold standard, *N*(1,2), assuming no missing at baseline. For example, if the missing rate is 10%, the cut point for determining missing data is 2.81.

A simulation was conducted to compare the proposed approach and popularly used missing data analyses mentioned in earlier sections (CC, LOCF, MI, EM, LMM, GEE). The proposed approach performed superior to the CC or LOCF under MCAR and performed better or similar to MI, EM, LMM or GEE regardless of the missing data mechanism. When MCAR or MAR, the B-spline model produced smaller biases with high coverage probabilities, however, it didn't show substantial difference under MNAR (Appendix I).

Appendix I

Simulation Results

Complete Case Analysis

Table 1 presents the results of three time-point models with the first order autoregressive correlation structure (AR(1)) for three scenarios of different missing rates when missing is MCAR. Under MCAR, the CC analysis produced a larger standard deviation and standard error than the proposed approach as the missing rate increased, while its coverage probability remained high due to the large standard deviation which was caused by discarding incomplete cases.

Table 1. Estimates of three parameter missing analyses with missing rates of (time1, time2, time3) under MCAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = 0, β_2(time$_{(3-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: autoregressive (1)

Missing rate	(10%, 10%, 10%)				(25%, 25%, 25%)				(10%, 25%, 50%)			
parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC												
β_0	0.0011	0.0516	0.0524	0.953	0.0014	0.0675	0.0686	0.955	0.0035	0.0764	0.0769	0.951
β_1	-0.0002	0.0411	0.0406	0.946	-0.0006	0.0531	0.0533	0.945	-0.0015	0.0606	0.0596	0.940
β_2	-0.0003	0.0531	0.0529	0.943	-0.0016	0.0699	0.0694	0.937	-0.0015	0.0788	0.0776	0.944
σ^2	0.0002	0.0531	NA	NA	-0.0083	0.0699	NA	NA	-0.0045	0.0788	NA	NA
ρ	-0.0002	0.0001	NA	NA	-0.0009	0.0003	NA	NA	-0.0005	0.0005	NA	NA
LOCF												
β_0	-0.0007	0.0455	0.0459	0.946	0.0007	0.0480	0.0480	0.946	0.0015	0.0458	0.0459	0.946
β_1	-0.0004	0.0349	0.0344	0.945	0.0002	0.0350	0.0342	0.946	-0.0006	0.0316	0.0314	0.957
β_2	-0.0004	0.0459	0.0452	0.942	-0.0009	0.0459	0.0451	0.949	-0.0005	0.0400	0.0395	0.952
σ^2	0.0025	0.0459	NA	NA	-0.0041	0.0459	NA	NA	-0.0007	0.0400	NA	NA
ρ	0.0192	0.0001	NA	NA	0.0533	0.0001	NA	NA	0.0984	0.0001	NA	NA
MI												
β_0	-0.0006	0.0457	0.0459	0.952	0.0029	0.0863	0.0445	0.743	0.0015	0.0542	0.0447	0.897
β_1	-0.0003	0.0385	0.0376	0.945	0.0021	0.0585	0.0489	0.898	0.0000	0.0538	0.0458	0.917
β_2	-0.0003	0.0491	0.0467	0.935	0.0009	0.0845	0.0535	0.811	0.0026	0.1499	0.0552	0.629
σ^2	0.0037	0.0491	NA	NA	-0.0147	0.0845	NA	NA	-0.0245	0.1499	NA	NA
ρ	-0.0362	0.0001	NA	NA	-0.2638	0.0003	NA	NA	-0.2990	0.0003	NA	NA
EM												
β_0	-0.0002	0.0554	0.0447	0.899	-0.0004	0.0514	0.0446	0.912	0.0015	0.0477	0.0447	0.926
β_1	-0.0006	0.0432	0.0415	0.948	0.0018	0.0459	0.0346	0.858	-0.0010	0.0433	0.0346	0.885
β_2	-0.0020	0.0578	0.0490	0.900	0.0005	0.0595	0.0450	0.862	-0.0015	0.0634	0.0450	0.840
σ^2	-0.0014	0.0578	NA	NA	-0.0058	0.0595	NA	NA	-0.0020	0.0634	NA	NA
ρ	-0.1053	0.0001	NA	NA	0.0001	0.0001	NA	NA	0.0007	0.0001	NA	NA
LMM												
β_0	-0.0006	0.0457	0.0459	0.952	0.0008	0.0489	0.0485	0.948	0.0014	0.0463	0.0460	0.943
β_1	-0.0003	0.0385	0.0376	0.945	0.0001	0.0437	0.0435	0.950	-0.0006	0.0401	0.0409	0.956
β_2	-0.0003	0.0491	0.0467	0.935	-0.0014	0.0552	0.0510	0.921	-0.0006	0.0593	0.0553	0.937
σ^2	0.0037	0.0491	NA	NA	-0.0030	0.0552	NA	NA	0.0011	0.0593	NA	NA
ρ	-0.0362	0.0001	NA	NA	-0.0614	0.0001	NA	NA	-0.0610	0.0001	NA	NA
GEE												
β_0	-0.0006	0.0467	0.0447	0.935	0.0009	0.0490	0.0492	0.954	0.0014	0.0464	0.0464	0.943
β_1	-0.0004	0.0408	0.0346	0.896	0.0001	0.0437	0.0433	0.951	-0.0006	0.0402	0.0404	0.952
β_2	-0.0005	0.0505	0.0451	0.923	-0.0014	0.0553	0.0543	0.941	-0.0005	0.0594	0.0589	0.956
σ^2	0.0016	0.0505	NA	NA	-0.0047	0.0553	NA	NA	-0.0009	0.0594	NA	NA
ρ	0.0004	0.0001	NA	NA	-0.0147	0.0002	NA	NA	-0.0128	0.0002	NA	NA
PROPOSED												
β_0	-0.0007	0.0457	0.0463	0.956	0.0006	0.0487	0.0488	0.947	0.0015	0.0460	0.0462	0.946
β_1	-0.0003	0.0385	0.0376	0.945	0.0002	0.0434	0.0428	0.946	-0.0006	0.0399	0.0402	0.959
β_2	-0.0003	0.0492	0.0485	0.945	-0.0012	0.0540	0.0530	0.950	-0.0007	0.0579	0.0575	0.953
σ^2	0.0019	0.0492	NA	NA	-0.0023	0.0540	NA	NA	0.0020	0.0579	NA	NA
ρ	-0.0055	0.0001	NA	NA	-0.0011	0.0001	NA	NA	-0.0008	0.0001	NA	NA

When the missing rate for each time point was 25% in Table 1, for example, the standard deviation and standard error of β_1 were 0.0531 and 0.0533 in CC analysis which is distant from 0.0434 and 0.0428 in the proposed approach. However, the coverage probabilities were close to 95% in both approaches. Besides, CC analysis retained above 94% of coverage probabilities in all parameters of interest even after 50% of data was missing. Hence, high coverage probability in CC analysis is not parallel to its accuracy. Table 1 shows that the proposed approach with AR(1) was superior to the CC analysis regarding both bias and coverage probability and it was remarked when missing rates were various for three time points (10%, 25%, 50%). The bias was -0.0015 in both β_1 and β_2 in CC analysis, however, the proposed approach produced less biases, -0.0006 and 0.0007 in β_1 and β_2, respectively. Also, coverage probabilities for estimates of β_1 and β_2 in proposed approach were higher than ones in CC analysis.

Imputation Approaches

When MCAR holds, the LOCF imputation method produced as small a bias as the proposed approach except for the correlation which was 0.0984 in LOCF and -0.0008 in the proposed when missing rates in three time points were 10, 25, 50% and also obtained high coverage probabilities above 94% across three different scenarios (Table 1). However, it yielded much smaller standard deviation and standard error compared to all other approaches. For example, when each time point had 25% of missing in Table 1, standard deviation and standard error of β_1 were 0.0350 and 0.0342, respectively, which are much smaller than 0.0434 and 0.0428 from the proposed approach and both standard deviation and standard error became smaller as missing rate increased due to replacement of missing data with latest previous observation. Nevertheless, the LOCF imputation method yielded comparable results to the proposed approach in terms of bias and coverage probability under MCAR.

Table 2. Estimates of three parameter missing analyses under MAR

Gold standard: β_0(Baseline) =1, β_1(time$_{(2-1)}$) = 0, β_2(time$_{(3-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: autoregressive (1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	0.0014	0.0433	0.0447	0.966	0.0012	0.0443	0.0447	0.950	0.0008	0.0429	0.0447	0.960
β_1	-0.1091	0.0393	0.0407	0.208	-0.1907	0.0500	0.0437	0.013	-0.4170	0.0851	0.0490	0.000
β_2	-0.1099	0.0477	0.0480	0.385	-0.1946	0.0576	0.0496	0.039	-0.4205	0.0957	0.0524	0.002
σ^2	-0.1262	0.0477	NA	NA	-0.1925	0.0576	NA	NA	-0.3122	0.0957	NA	NA
ρ	-0.1104	0.0001	NA	NA	-0.1829	0.0002	NA	NA	-0.3438	0.0003	NA	NA
EM												
β_0	0.0014	0.0433	0.0447	0.966	0.0012	0.0443	0.0447	0.950	0.0008	0.0429	0.0447	0.960
β_1	-0.0090	0.0347	0.0348	0.937	-0.0160	0.0391	0.0350	0.905	-0.0459	0.0483	0.0353	0.676
β_2	-0.0176	0.0454	0.0456	0.931	-0.0379	0.0493	0.0460	0.848	-0.1322	0.0595	0.0473	0.254
σ^2	-0.0289	0.0454	NA	NA	-0.0562	0.0493	NA	NA	-0.1243	0.0595	NA	NA
ρ	-0.0098	0.0001	NA	NA	-0.0201	0.0001	NA	NA	-0.0502	0.0001	NA	NA
LMM												
β_0	0.0121	0.0439	0.0442	0.942	0.0180	0.0450	0.0438	0.926	0.0251	0.0435	0.0432	0.918
β_1	-0.0280	0.0348	0.0358	0.882	-0.0472	0.0383	0.0370	0.745	-0.1013	0.0428	0.0407	0.315
β_2	-0.0556	0.0467	0.0458	0.769	-0.0983	0.0498	0.0467	0.456	-0.2162	0.0570	0.0500	0.018
σ^2	-0.0430	0.0467	NA	NA	-0.0725	0.0498	NA	NA	-0.1215	0.0570	NA	NA
ρ	-0.0410	0.0001	NA	NA	-0.0511	0.0001	NA	NA	-0.0710	0.0001	NA	NA
GEE												
β_0	0.0098	0.0437	0.0452	0.958	0.0128	0.0447	0.0452	0.941	0.0157	0.0434	0.0451	0.953
β_1	-0.0321	0.0349	0.0361	0.865	-0.0588	0.0386	0.0374	0.657	-0.1373	0.0435	0.0409	0.096
β_2	-0.0639	0.0466	0.0479	0.740	-0.1180	0.0495	0.0496	0.331	-0.2657	0.0554	0.0534	0.001
σ^2	-0.1215	0.0466	NA	NA	-0.1775	0.0495	NA	NA	-0.2411	0.0554	NA	NA
ρ	-0.0801	0.0001	NA	NA	-0.1185	0.0001	NA	NA	-0.1662	0.0001	NA	NA
PROPOSED												
β_0	0.0014	0.0433	0.0432	0.959	0.0012	0.0443	0.0425	0.939	0.0008	0.0429	0.0415	0.943
β_1	-0.0124	0.0343	0.0365	0.943	-0.0263	0.0379	0.0379	0.904	-0.0873	0.0423	0.0425	0.475
β_2	-0.0229	0.0450	0.0468	0.932	-0.0516	0.0481	0.0482	0.822	-0.1647	0.0541	0.0525	0.127
σ^2	-0.1378	0.0450	NA	NA	-0.1974	0.0481	NA	NA	-0.2758	0.0541	NA	NA
ρ	-0.0450	0.0001	NA	NA	-0.0712	0.0001	NA	NA	-0.1245	0.0001	NA	NA

Table 3. Estimates of four parameter missing analyses under MAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure : autoregressive (1)

Missing rate	(group1, group2) = (5%, 10%)				(group1, group2) = (10%, 25%)			
Variable	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0083	0.0556	0.0530	0.926	0.0169	0.0546	0.0528	0.933
β_1	-0.0417	0.0608	0.0584	0.884	-0.0790	0.0635	0.0594	0.716
β_2	-0.0443	0.0688	0.0621	0.863	-0.0837	0.0779	0.0623	0.686
β_3	-0.0149	0.0579	0.0564	0.934	-0.0350	0.0640	0.0550	0.853
σ^2	-0.0165	0.0561	NA	NA	-0.0225	0.0676	NA	NA
ρ	-0.0598	0.0256	NA	NA	-0.0972	0.0314	NA	NA
EM								
β_0	0.0018	0.0552	0.0534	0.939	0.0023	0.0531	0.0534	0.951
β_1	-0.0014	0.0617	0.0566	0.929	-0.0035	0.0662	0.0568	0.905
β_2	-0.0072	0.0683	0.0621	0.924	-0.0205	0.0693	0.0625	0.908
β_3	-0.0021	0.0569	0.0583	0.955	-0.0058	0.0556	0.0579	0.950
σ^2	-0.0059	0.0561	NA	NA	-0.0062	0.0589	NA	NA
ρ	-0.0027	0.0280	NA	NA	-0.0122	0.0315	NA	NA
LMM								
β_0	0.0081	0.0555	0.0534	0.932	0.0127	0.0536	0.0537	0.947
β_1	-0.0133	0.0601	0.0586	0.938	-0.0290	0.0623	0.0614	0.927
β_2	-0.0395	0.0677	0.0626	0.873	-0.0720	0.0673	0.0646	0.788
β_3	-0.0079	0.0573	0.0590	0.953	-0.0186	0.0572	0.0603	0.955
σ^2	-0.0093	0.0536	NA	NA	-0.0112	0.0553	NA	NA
ρ	-0.0420	0.0347	NA	NA	-0.0637	0.0380	NA	NA
GEE								
β_0	0.0083	0.0554	0.0537	0.931	0.0135	0.0535	0.0540	0.948
β_1	-0.0160	0.0602	0.0583	0.940	-0.0357	0.0623	0.0609	0.914
β_2	-0.0427	0.0677	0.0642	0.877	-0.0780	0.0665	0.0664	0.780
β_3	-0.0093	0.0571	0.0586	0.953	-0.0218	0.0571	0.0597	0.943
σ^2	-0.0168	0.0531	NA	NA	-0.0187	0.0549	NA	NA
ρ	-0.0454	0.0219	NA	NA	-0.0694	0.0239	NA	NA
PROPOSED								
β_0	0.0029	0.0555	0.0533	0.933	0.0058	0.0536	0.0536	0.947
β_1	-0.0070	0.0599	0.0587	0.942	-0.0200	0.0621	0.0614	0.940
β_2	-0.0128	0.0668	0.0633	0.928	-0.0326	0.0660	0.0655	0.916
β_3	-0.0043	0.0579	0.0585	0.949	-0.0127	0.0579	0.0596	0.960
σ^2	-0.0156	0.0532	NA	NA	-0.0172	0.0550	NA	NA
ρ	-0.0291	0.0240	NA	NA	-0.0472	0.0278	NA	NA

The MI method produced a larger bias, standard deviation and standard error but lower coverage probability when missing data is MCAR (Table 1). When each time point had 25% of missing in Table 1, the bias of β_1 in MI was 0.0021 which is ten times larger than one from the proposed approach, indeed, the coverage probability was 89.8% which is 5% less than the proposed one. Besides, the standard deviation was far apart from the standard error and the difference became larger as missing rate increased, leading to a lower coverage probability. The MI also showed similarities in larger bias and great difference between standard deviation and standard error as well as inferior coverage probability under MAR (Table 2 and 3). In Table 2, for example of β_1, the MI yielded -0.4170 in bias leading 0% of coverage probability when the missing rate was 25% which is much largely biased than one from the proposed approach, -0.0873 with 47.5% of coverage probability. Besides, the difference between standard deviation and standard error in MI was 0.0360, however, it was 0.0002 in the proposed approach. When we separated the data by group and had low true correlation (ρ = 0.2), the MI still produced larger biases and lower coverage probabilities than the proposed approach (Table 3). Overall, the proposed approach performed better than the MI regardless of missing data mechanism or correlation structure (Appendix).

Under MCAR, the EM algorithm approach produced larger bias compared to the proposed approach especially if AR(1), besides, it also yielded a larger difference between standard deviation and standard error causing lower coverage probability in both correlation structures (Table 1). When each time point had 25% of missing in Table 1, the bias of β_1 was 0.0018 in the EM algorithm approach while it was 0.0002 in the proposed one. The EM algorithm approach also retained 85.8% of coverage probability which is lower than the proposed one, 94.6%. Also, the difference between standard deviation and standard error in the EM algorithm approach was 0.0113 which is much larger than 0.0006 in the proposed one. Although it produced smallest bias and highest coverage probability under MAR, substantial difference between standard deviation and standard error remained in problem. With 25% of missing rate, the difference between standard deviation and standard error in the EM algorithm approach was 0.0122 which is much larger than 0.0016 in the proposed one (Table 2).

Under MNAR, the MI method showed greater bias and inferior coverage probability and the EM algorithm approach yielded similar results compared to the proposed approach, yet both showed a larger difference between standard deviation and standard error (Appendix).

Comparisons among Approaches using all Available Data

The LMM behaved very similar to the proposed approach under MCAR, except that it obtained the substantially biased correlation estimate (Table 1). When each time point had 10% of missing in Table 1, the bias of correlation in the LMM was -0.0362 while the proposed one yielded -0.0055.When MAR holds, the proposed approach with AR(1) performed superior to the LMM. With 10% of missing rate in Table 2, the LMM produced -0.0472 and -0.0983 in bias of β_1 and β_2, respectively, which are larger than -0.0263 and -0.0516 from the proposed approach. Also, the coverage probabilities in the proposed approach were 90.4% and 82.2% for β_1 and β_2, respectively, which is much higher than ones from LMM, 74.5% and 45.6%. Furthermore, the proposed approach remarkably performed better in terms of bias and coverage probability when the true correlation parameter was low ($\rho = 0.2$) (Table 3). With missing rates of group1 and group2 were 10 and 25% in Table 3, the LMM yielded -0.0720 in bias of β_2 which is larger than -0.0326 from the proposed approach and the coverage probability was also lower in the LMM (78.8%) than the proposed one (91.6%). Besides, the difference between standard deviation and standard error was smaller in the proposed approach. The results showed the similarities between the LMM and the proposed approach under MNAR, however, estimates from the proposed approach with AR(1) tended to be less biased (see Appendix (Table 9.1-9.4)).

The GEE approach also showed analogous results to the proposed one under MCAR except for the correlation which was much less biased in the proposed one (Table 1). With 25% missing rate of each time point in Table 1, the GEE approach produced -0.0147 in bias of correlation while the proposed one produced -0.0011. When MAR holds, the proposed approach performed better than the GEE approach, especially if correlation structure is AR(1) (Table 2). When the missing rate was 10% in Table 2, the GEE approach yielded -0.0588 and -0.1180 in bias of β_1 and β_2, respectively, which are much larger than -0.0263 and -0.1974 from the proposed one. Also, the coverage probabilities in the GEE approach (65.7 and 33.1%) were also much lower than ones in the proposed approach (90.4 and 82.2%). Under MNAR, both the GEE and proposed approach produced similar results, however, the proposed one tended to attain smaller bias and higher coverage probability (Appendix (Table 9.1-9.4)).Overall, the proposed approach performed superior to the GEE approach if the correlation structure is AR(1) regardless of the missing data mechanism.

The random-effect pattern mixture model yielded a smaller bias on time effect (β_1), yet it produced greater biases for other estimates (Table 4). When $\rho = 0.7$ in Table 4, a bias of β_1 in the random-effect pattern mixture model was -0.0919 which is smaller than both LMM (-0.1644) and the proposed approach (-0.1734), however, β_2 was much more biased (-0.1842) than LMM (-0.0480) and the proposed one (-0.0508). Hence, the random-effect pattern mixture model does not seem to be a compatible alternative over the proposed approach under MNAR mechanism unless time effect itself becomes the primary interest in a study.

B-Spline

When MCAR or MAR, the B-spline model produced smaller biases with high coverage probabilities except for the baseline (β_0) and its standard deviation and error became also much smaller when B-spline was applied (Table 5).When MAR with 25% of missing in Table 5, the bias and standard deviation of β_3 were reduced from -0.3018 to 0.0014 and 0.3465 to 0.0773, respectively. The results did not show much difference among three B-spline models with different number of knots which means that adding more knots did not improve the model fitting in this example. Under MNAR, B-spline model did not make a substantial difference compared to the model without B-spline (Appendix II).

Applications

Data

The TIME Trial is a phase II, randomized, placebo-controlled clinical trial, conducted by the Cardiovascular Cell Therapy Research Network (CCTRN) to assess the safety, effect and most efficient timing of bone marrow mononuclear cell (BMMNC) therapy after an Acute Myocardial Infarction (AMI) (Traverse *et al*., 2009).

Table 4. Estimates of five parameter missing analyses under MNAR with missing rates in (group11, group12, group21, group22) = (5%, 10%, 10%, 25%)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(group$_{(21-11)}$) = β_3(group$_{(22-12)}$) = 0, σ^2 = 2, Correlation structure: Exchangeable

Correlation	ρ = 0.7				ρ = 0.2			
Variable	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
LMM								
β_0	0.0716	0.4524	0.4461	0.941	0.0988	0.3802	0.3724	0.941
β_1	-0.1644	0.0365	0.0364	0.003	-0.3049	0.0556	0.0559	0.000
β_2	-0.0480	0.0774	0.0784	0.905	-0.0937	0.0634	0.0653	0.710
β_3	-0.0493	0.0738	0.0784	0.926	-0.0958	0.0664	0.0653	0.684
β_4	-0.0005	0.0099	0.0098	0.946	-0.0001	0.0082	0.0082	0.951
σ^2	-0.2031	0.0706	NA	NA	-0.2706	0.0583	NA	NA
ρ	-0.1519	0.0217	NA	NA	-0.1583	0.0201	NA	NA
Random Effect Pattern Mixture								
β_0	-0.0333	0.4538	0.4294	0.933	0.0595	0.3902	0.3852	0.945
β_1	-0.0919	0.0363	0.0361	0.291	-0.2484	0.0572	0.0572	0.009
β_2	-0.1842	0.0751	0.0756	0.334	-0.1284	0.0645	0.0677	0.518
β_3	-0.1858	0.0744	0.0756	0.297	-0.1314	0.0673	0.0677	0.490
β_4	-0.0003	0.0100	0.0094	0.938	0.0000	0.0084	0.0084	0.948
σ^2	-0.4733	0.0553	NA	NA	-0.2895	0.0582	NA	NA
ρ	-0.2076	0.0318	NA	NA	-0.1597	0.0200	NA	NA
PROPOSED								
β_0	0.0741	0.4513	0.4326	0.930	0.0987	0.3794	0.3717	0.941
β_1	-0.1734	0.0367	0.0377	0.002	-0.3051	0.0556	0.0560	0.000
β_2	-0.0508	0.0771	0.0760	0.891	-0.0938	0.0634	0.0652	0.709
β_3	-0.0521	0.0735	0.0760	0.906	-0.0958	0.0663	0.0652	0.684
β_4	-0.0005	0.0099	0.0095	0.940	-0.0001	0.0082	0.0081	0.951
σ^2	-0.2743	0.0633	NA	NA	-0.2720	0.0575	NA	NA
ρ	-0.0687	0.0201	NA	NA	-0.0371	0.0315	NA	NA

Table 5. Estimates of five parameter proposed approach with spline assuming the same missing rate in each group

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(group$_{(21-11)}$) = β_3 (group$_{(22-12)}$) = β_3(continuous) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

	MCAR				MAR			
	10%		25%		10%		25%	
Missing rate	Bias	Std.Dev	Bias	Std.Dev	Bias	Std.Dev	Bias	Std.Dev
PROPOSED								
β_0	0.0417	0.0680	0.0417	0.0729	0.0254	0.1879	0.0251	0.1950
β_1	-0.0003	0.0432	-0.0022	0.0476	-0.0301	0.0507	-0.0720	0.0899
β_2	-0.0046	0.0871	-0.0021	0.0863	-0.0006	0.0811	0.0003	0.0826
β_3	-0.5284	0.1052	-0.5815	0.1130	-0.2734	0.3107	-0.3018	0.3465
β_4	0.2799	0.0294	0.3095	0.0336	0.1431	0.1574	0.1581	0.1737
σ^2	-0.3542	0.0644	-0.3427	0.0705	-0.2104	0.2283	-0.2269	0.2348
ρ	-0.1740	0.0264	-0.1737	0.0318	-0.1282	0.1346	-0.1538	0.156
PROPOSED_splines (3 knots)								
β_0	-2.8443	0.5174	-2.8138	0.5471	-1.2845	1.5170	-1.3017	1.5117
β_1	0.0009	0.0387	-0.0001	0.0435	-0.0023	0.0371	0.0041	0.0482
β_2	-0.0045	0.0645	0.0001	0.0676	-0.0003	0.0743	0.0018	0.0738
β_3	-0.0042	0.0633	0.0018	0.0662	-0.0026	0.0758	0.0014	0.0773
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.5230	0.0583	-0.5206	0.0635	-0.3326	0.356	-0.3597	0.3721
ρ	-0.0220	0.0199	-0.0016	0.0222	-0.0643	0.0678	-0.077	0.0769
PROPOSED_splines (5 knots)								
β_0	-3.1965	0.6969	-3.1901	0.7580	-1.4636	1.7749	-1.4855	1.7816
β_1	0.0009	0.0387	-0.0001	0.0435	-0.0023	0.0371	0.0038	0.0481
β_2	-0.0043	0.0644	0.0001	0.0674	-0.0003	0.0743	0.0019	0.0739
β_3	-0.0044	0.0634	0.0020	0.0662	-0.0024	0.0757	0.0014	0.0773
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.5263	0.0586	-0.5219	0.0638	-0.3334	0.3568	-0.3613	0.3738
ρ	-0.0233	0.0200	-0.0021	0.0223	-0.0647	0.0682	-0.0784	0.0783
PROPOSED_splines (7 knots)								
β_0	-3.2227	0.7584	-3.2177	0.8588	-1.4769	1.8280	-1.4996	1.8307
β_1	0.0009	0.0387	-0.0001	0.0435	-0.0022	0.0371	0.0039	0.0481
β_2	-0.0045	0.0645	0.0001	0.0675	-0.0003	0.0744	0.0018	0.0739
β_3	-0.0043	0.0634	0.0020	0.0661	-0.0025	0.0757	0.0016	0.0775
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.5267	0.0586	-0.5223	0.0637	-0.3335	0.3569	-0.3614	0.374
ρ	-0.0237	0.0201	-0.0025	0.0223	-0.0649	0.0683	-0.0786	0.0785

TIME has two objectives: (a) to evaluate the effect of a single intracoronary infusion of autologous BMMNC on global LV function compared to a control infusion of 5% human serum albumin and (b) to assess the most efficient timing of therapy administration, 3 or 7 days after AMI. Global LV function was examined by cardiac Magnetic Resonance Imaging (cMRI) at Study Production Infusion (SPI) time (Day3/Day7) and 6 months and the changes at 6 month from SPI time were obtained as a primary outcome. LV myocardial mass, volume, ejection fraction and regional systolic wall motion, thickening, radial displacement in the infarct and border zone were measured to assess global LV function, which was assumed to be normally distributed for the analysis in this study.

A total of 120 post-AMI patients who were satisfied inclusion and exclusion criteria for TIME were enrolled from 5 sites and randomized to treatment and control therapy in a ratio of 2:1 for each time administration (Traverse *et al*., 2009). Study population was adults at least 21 years of age.

Results

The distribution of population in TIME study was summarized in the previous article (Traverse *et al*., 2012). There were substantially more males than females in the randomized cohort. Patients were on average 57 years of age. Cell therapy produced no significant changes in LV function in either the Day 3 or Day 7 evaluations. There were no missing values at SPI, but four missing occurred at the Day 3 endpoint and two and three missing values occurred at SPI and endpoint, respectively on Day7. Hence, the total missing rate was 3.0% on Day3 and 4.7% on Day7.

Table 6 showed the results of the effect of the model adjusted for gender and age, separately by time administrations (Day3 and Day7). The BMMNC therapy effect did not show a large difference from the basic model except for MI and EM algorithm approach which demonstrated fluctuations after the adjustment. For example on Day7, the point estimate of BMMNC therapy was decreased 0.33% after adjusting for gender and age in proposed approach, while it was decreased 15.49% in MI and 43.33% in EM algorithm approach. It showed that imputation methods may be more sensitive on covariates included in the model as adjustments than the methods which incorporate all available data. The time effect was slightly changed after the adjustment, yet none of the 7 missing analyses applied (CC, LOCF, MI, EM, LMM, GEE) including the proposed approach showed a change on its significance. Global LV function among female was significantly larger than male on Day3, yet not on Day7. Age did not show statistical significance on improvement of global LV function. The absolute effect size of BMMNC therapy increased after adjustment for gender and age in both time administrations but not significantly.

A scatter plot of age and the global LV function at SPI time are presented in Fig. 1 with the predicted lines (solid line) from B-spline models with eight and five internal knots for Day3 and Day7, respectively. The number of internal knots was determined based on the improvement of the model fitting by the spline. The two dashed lines from each plot represent the upper and lower 95% confidence intervals. Table 7 presents the results of the proposed approach without and with B-spline function. The mean global LV function at SPI changed from 43.150 to 41.964 on Day3 and from 48.640 to 51.202 on Day7 when B-spline function was applied. B-spline moved the BMMNC therapy effect towards the null value in both time administrations, changing from 4.447 to 4.0 on Day3 and from -4.629 to -3.743 on Day7. However, other effects showed analogous results between models without and with B-spline.

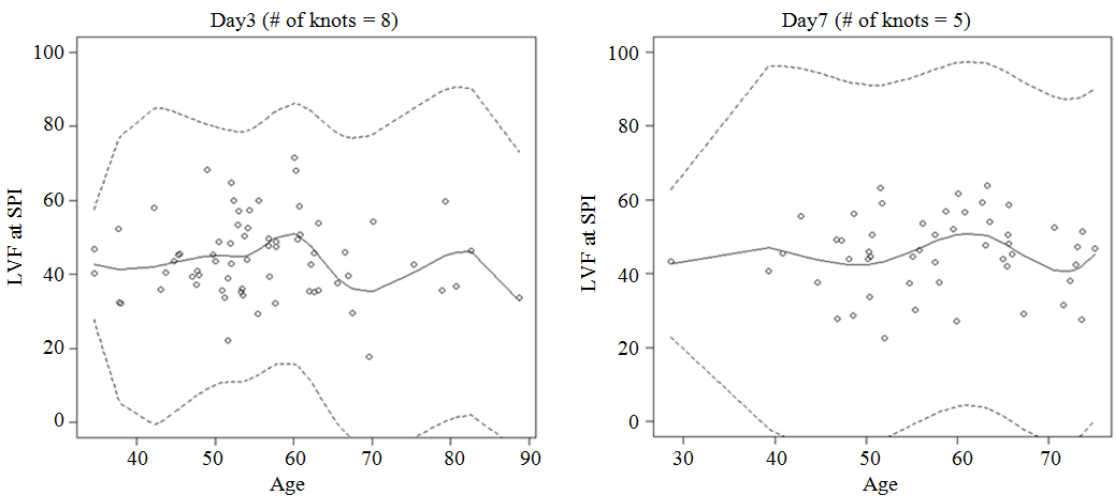

Fig. 1. Plot of age and the global left ventricular function at SPI time with the predicted line and its 95% confidence intervals from the model with splines

Table 6. TIME parameter estimates from seven missing analyses in the basic model adjusting for gender and age at each time administration (Day3/Day7)

Variable	PROPOSED	LMM	GEE	CC	LOCF	MI	EM
Day3							
LVF at SPI							
(95% CI)	43.150	43.148	43.148	42.628	43.319	43.024	43.793
	(29.846, 56.454)	(29.904, 56.392)	(31.747, 54.549)	(30.925, 54.331)	(31.986, 54.652)	(31.654, 54.394)	(32.266, 55.320)
BMMNC							
(95% CI)	4.447	4.446	4.445	4.223	4.571	4.493	4.149
	(-0.727, 9.621)	(-0.705, 9.597)	(-0.537, 9.427)	(-1.024, 9.470)	(-0.358, 9.500)	(-0.434, 9.420)	(-0.953, 9.251)
Time							
(95% CI)	3.728	3.727	3.727	3.799	3.572	3.778	4.519
	(1.107, 6.349)	(1.095, 6.359)	(1.089, 6.365)	(1.161, 6.437)	(1.083, 6.061)	(1.248, 6.308)	(1.906, 7.132)
Gender							
(95% CI)	14.194	14.195	14.195	14.340	14.283	14.168	13.736
	(6.607, 21.781)	(6.643, 21.747)	(7.057, 21.333)	(7.145, 21.535)	(7.147, 21.419)	(7.043, 21.293)	(6.506, 20.966)
Age							
(95% CI)	-0.057	-0.057	-0.057	-0.048	-0.061	-0.055	-0.064
	(-0.277, 0.163)	(-0.277, 0.163)	(-0.251, 0.137)	(-0.246, 0.15)	(-0.255, 0.133)	(-0.249, 0.139)	(-0.260, 0.132)
σ^2	133.874	133.152	126.937	128.716	125.038	125.548	135.262
ρ	0.574	0.508	0.565	0.557	0.568	0.555	0.560
Day7							
LVF at SPI							
(95% CI)	48.640	48.644	48.640	49.422	48.365	48.232	46.745
	(31.772, 65.508)	(31.935, 65.353)	(35.943, 61.337)	(36.547, 62.297)	(35.633, 61.097)	(34.628, 61.836)	(33.072, 60.418)
BMMNC							
(95% CI)	-4.629	-4.626	-4.629	-4.147	-4.661	-4.497	-5.746
	(-10.911, 1.653)	(-10.847, 1.595)	(-10.811, 1.553)	(-10.676, 2.382)	(-10.790, 1.468)	(-10.843, 1.849)	(-12.055, 0.563)
Time							
(95% CI)	2.418	2.418	2.418	2.419	2.309	2.025	2.585
	(-0.106, 4.942)	(-0.13, 4.966)	(-0.112, 4.948)	(-0.125, 4.963)	(-0.127, 4.745)	(-0.415, 4.465)	(0.131, 5.039)
Gender							
(95% CI)	1.597	1.596	1.597	1.504	1.652	1.831	0.922
	(-6.817, 10.011)	(-6.738, 9.930)	(-3.911, 7.105)	(-4.100, 7.108)	(-3.848, 7.152)	(-3.733, 7.395)	(-4.605, 6.449)
Age							
(95% CI)	-0.004	-0.004	-0.004	-0.022	0.001	0.000	0.052
	(-0.290, 0.282)	(-0.288, 0.280)	(-0.210, 0.202)	(-0.232, 0.188)	(-0.207, 0.209)	(-0.233, 0.233)	(-0.179, 0.283)
σ^2	131.817	130.077	122.554	125.630	120.897	128.682	126.565
ρ	0.689	0.621	0.689	0.671	0.680	0.680	0.672

Table 7. Estimates from the proposed approach in the basic model adjusting for gender and age with and without spline using generalized least squares method at each time administration (Day3/Day7)

	Day3		Day7	
Variable	without spline	with spline	without spline	with spline
LVF at SPI				
(95% CI)	43.150	41.964	48.640	51.202
	(29.846, 56.454)	(27.452, 56.476)	(31.772, 65.508)	(29.46, 72.944)
BMMNC				
(95% CI)	4.447	4.000	-4.629	-3.743
	(-0.727, 9.621)	(-1.151, 9.151)	(-10.911, 1.653)	(-10.36, 2.874)
Time				
(95% CI)	3.728	3.724	2.418	2.460
	(1.107, 6.349)	(1.043, 6.405)	(-0.106, 4.942)	(0.069, 4.851)
Gender				
(95% CI)	14.194	14.129	1.597	1.551
	(6.607, 21.781)	(6.714, 21.544)	(-6.817, 10.011)	(-6.965, 10.067)
Age				
(95% CI)	-0.057	NA	-0.004	NA
	(-0.277, 0.163)	NA	(-0.29, 0.282)	NA
σ^2	133.874	127.923	131.817	129.691
ρ	0.574	0.533	0.689	0.716

Discussion

Public health relies on clinical trials to approve an effect of therapy or drug. However, clinical trial interpretation is vitiated in the presence of missing data, a common occurrence in clinical trials. Since missing outcome in longitudinal studies occurred more frequently, it is important for statisticians to perform the most compatible longitudinal data analysis in missing outcome environment.

The proposed approach based on the generalized least squares method produces the best linear unbiased estimate if the covariance matrix V is known even in the presence of missing outcome. In practice, however, V is usually unknown. In the proposed approach, the REML method was applied to estimate the correlation parameter assuming common variance σ^2 under the pre-specified correlation structure (i.e., compound symmetry or AR(1)) using all paired data. Since only all paired data is used for parameter estimation of correlation, it maylead a bias if all paired data does not represent the entire sample in terms of the correlation over time within subjects. However, this is not a problem of only the proposed approach, all other missing analyses incorporating all available data, e.g., LMM and the GEE approach, also have difficulties of estimating covariance parameters in fractional data. The problem can be even worse in imputation methods due to the additional uncertainty by imputing a value which is not actually observed.

Another limitation of the proposed approach is that it only allows time-invariant covariates, hence, further research is required for managing the time-variant covariates. Nevertheless, the simulation study showed the validity of the proposed approach, especially its superiority where the correlation structure is AR(1) under MAR mechanism which is a more plausible in longitudinal health research. The proposed approach also can be compatible when missing data is MCAR regardless of correlation structure, however, it may not be valid under MNAR.

Each missing analysis applied in this study has critical pitfalls along with strengths and benefits. The CC analysis is easy to execute which is a great advantage over the proposed approach, however, it obtained a larger variance due to loss of information which can be drastic in longitudinal setting. Also, CC analysis can be seriously biased unless MCAR holds (Demissie *et al*., 2003). Imputations can be useful when the conditions are met, however, it can also lead a severe bias, inflating the uncertainty of parameter estimate by imputing unobserved value. Simple imputations such as LOCF are also easy to carry out, yet, it ignores the variability of missing data which is not plausible in reality.

Also, it addresses a serious bias unless MCAR holds (Lane, 2008). The MI reflects the variability of missing data complementing the drawback of simple imputations, nevertheless, its plausibility is still vague (Sterne *et al*., 2009; Engels and Diehr, 2003; Schafer and Olsen, 1998). Besides, since it is a distribution-based method, it can lead a bias if the distribution is mis-defined, while the proposed approach is not restricted to the distribution since it is based on the Gauss-Markov theorem. The EM algorithm is a likelihood-based imputation method guaranteed to maximize the likelihood; however, it also has shortcomings that the proposed approach does not have, e.g., not guaranteed to obtain the closed form which is an analytical expression having finite number of functions, convergence issue. Moreover, both MI and the EM algorithm approach, showed substantial difference between standard deviation and standard error indicating unreliable estimate in the simulation study. This was caused by the additional uncertainty from imputed values which were not actually observed.

Missing analyses incorporating all available data behaved similar to the proposed approach. The LMM, yielding BLUE for fixed effects when the covariance is known, allows unbalanced data when MAR holds (Beunckens *et al*., 2005; Laird and Ware, 1982). However, it can fail to converge for several reasons such as ill-conditioned samples, mis-specified model or violation of the normality assumption. Also, the models are more restricted due to additional assumptions for random effects. Therefore, the proposed approach can be widely used in various situations than LMM since it is less constrained in assumptions. The GEE approach is less restricted by employing the estimating equations with working correlation instead of using fully joint multivariate distribution (Liang and Zeger, 1986), yet, it produces inconsistent estimate unless MCAR holds. Robin (Robins *et al*., 1995) developed the weighted GEE yielding consistent estimates under MAR. However, it is only valid in a correctly specified model with consistent missing probability, besides, its superiority is still vague (Qu *et al*., 2011). We applied the random-effect pattern mixture model for the comparison to the proposed approach when missing is MNAR. The random-effect pattern mixture model was not superior to the proposed approach unless time effect is the primary interest in the study. The purpose of the simulation study is to understand some of the asymptotic properties of the estimates, hence, it requires large sample sizes. Estimator properties in small sample sizes are worthy of further examination. In addition, it may not be valid unless it has sufficient number of missing pattern. However, there are several other methods introduced for managing missing not at random, further research can be made on comparing them to the proposed approach (Troxel *et al*., 1998).

The simulation study showed that applying B-spline can help in obtaining smaller variance of estimate which derives the efficient hypothesis testing under MCAR or MAR. However, it didn't show much improvement on the model fitting under MNAR, hence, further research may be required for the evaluation.

Although most missing data analyses are restricted to the missing data mechanism, it is difficult to identify the missing mechanism (MCAR or MAR) in reality. Hence, it is hard to determine which missing analysis is the most compatible in TIME trial. Besides, with small number of missing data in TIME trial, missing analyses may not be quite distinguishable on BMMNCs therapy effect. However, we expect more variability among missing analyses in future studies with higher missing rates since loss of information or adding more uncertainty to the estimates by imputations tends to carry a greater risk of contorting the truth than approaches using all available data. Although the LMM or GEE approach also incorporates all available data, the proposed approach is less restricted than the LMM since it does not include random effects and more flexible than GEE approach regarding the missing mechanism. Therefore, the proposed approach can be an alternative of current missing analysis with a continuous outcome.

There is no perfect solution for missing data problems due to loss of information which should have been collected. Although missing information can be categorized, it is difficult to identify three types of missing data mechanism in practice, except for a few cases where MCAR and MAR are distinguishable. Also, current models are properly established with complete data, hence, none can promise the correct solution in missing environment. Therefore, we should first and foremost try our best to collect complete data. In the presence of missing, we need to fully comprehend the study and apply the most compatible approach. We believe that the proposed approach can be a powerful alternative for managing missing outcomes in longitudinal studies where the correlation over time within subjects can be expressed as AR(1).

Conclusion

Since missing data is commonly occurred in health research, many statistical methodologies have been introduced to effectively manage missing data problems. Although more advanced methods are recommended, simple approaches still have been used for the primary analysis in majority of studies due to its simplicity in understanding and execution. The proposed approach is as straightforward as simple approaches, incorporating all available data without deletion or imputation of information. The simulation study showed its validity, especially it was superior to current missing analyses applied for the comparison when the correlation structure was the first-order autoregressive under missing at random mechanism. The proposed approach was also applied to the Transplantation in Myocardial Infarction Evaluation trial. It produced similar results to the linear mixed model or generalized estimating equation approach, however, distinct results from all other approaches involved in deletion or imputation. The proposed approach may lead to bias if missing data is not missing at random. This is because the correlation parameter is estimated from all paired data when the true parameter is unknown. Nevertheless, our approach can be a useful alternative of current missing analyses. It avoids either to waste information or to add more uncertainty to the estimate by imputation, besides, it is less restricted by not including additional terms to the model and also flexible on the missing data mechanism compared to the generalized estimating equation approach. The proposed approach in this study considers only time-invariant covariates, hence, further study managing time-variant covariate is required since time-variant variable become an important confounder sometimes in health research. Also, extending the model with binary or categorical outcome can be considered in the future study.

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Author's Contributions

Minjeong Park and Lemuel Moye: Contributed to the concept and design of the study, data analyses, interpretation of results and draft and approval of this manuscript.

Dejian Lai, Xianglin Du, George Delclos: Contributed to the degisn of the study, data analyses, interpretation of results, draft and approval of this manuscript.

Ethics

This study has been approved by the Committee for the Protection of Human Subjects at the University of Texas Health Science Center. The authors declare that they have no conflict of interest.

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Appendix II. Tables of Results from Simulation Study

Table 1.1. Estimates of three parameter missing analyses under MCAR with missing rates of (time1, time2, time3)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.7$ Correlation structure: Exchangeable

Missing rate	(10%, 10%, 10%)				(25%, 25%, 25%)				(10%, 25%, 50%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC												
β_0	-0.0016	0.0510	0.0497	0.941	-0.0011	0.0604	0.0596	0.943	-0.0015	0.0731	0.0729	0.941
β_1	0.0018	0.0382	0.0384	0.947	0.0014	0.0448	0.0462	0.959	-0.0003	0.0553	0.0565	0.955
β_2	0.0019	0.0394	0.0385	0.939	0.0002	0.0444	0.0462	0.961	-0.0015	0.0569	0.0565	0.953
σ^2	-0.0009	0.0394	NA	NA	-0.0012	0.0444	NA	NA	-0.0064	0.0569	NA	NA
ρ	-0.0002	0.0001	NA	NA	-0.0004	0.0001	NA	NA	-0.0009	0.0002	NA	NA
LOCF												
β_0	-0.0019	0.0448	0.0447	0.951	0.0007	0.0452	0.0447	0.938	-0.0036	0.0466	0.0448	0.939
β_1	0.0014	0.0322	0.0328	0.956	0.0002	0.0296	0.0300	0.948	0.0004	0.0290	0.0300	0.948
β_2	0.0016	0.0349	0.0345	0.944	-0.0005	0.0323	0.0335	0.964	0.0000	0.0330	0.0324	0.944
σ^2	-0.0017	0.0349	NA	NA	0.0009	0.0323	NA	NA	0.0004	0.0330	NA	NA
ρ	0.0207	0.0001	NA	NA	0.0561	0.0001	NA	NA	0.0874	0.0001	NA	NA
MI												
β_0	-0.0019	0.0448	0.0447	0.951	0.0007	0.0452	0.0447	0.938	-0.0036	0.0466	0.0448	0.939
β_1	0.0024	0.0411	0.0384	0.938	0.0006	0.0655	0.0435	0.851	0.0052	0.0645	0.0435	0.851
β_2	0.0019	0.0484	0.0384	0.886	0.0012	0.0836	0.0435	0.746	0.0104	0.1596	0.0505	0.560
σ^2	-0.0021	0.0484	NA	NA	-0.0048	0.0836	NA	NA	-0.0209	0.1596	NA	NA
ρ	-0.0902	0.0001	NA	NA	-0.2160	0.0002	NA	NA	-0.3139	0.0003	NA	NA
EM												
β_0	-0.0019	0.0448	0.0447	0.951	0.0007	0.0452	0.0447	0.938	-0.0036	0.0466	0.0448	0.939
β_1	0.0018	0.0364	0.0346	0.932	0.0001	0.0413	0.0346	0.894	0.0005	0.0408	0.0346	0.904
β_2	0.0017	0.0371	0.0346	0.932	-0.0011	0.0407	0.0347	0.908	-0.0004	0.0520	0.0346	0.812
σ^2	-0.0008	0.0371	NA	NA	0.0014	0.0407	NA	NA	-0.0001	0.0520	NA	NA
ρ	-0.0001	0.0001	NA	NA	-0.0002	0.0001	NA	NA	0.0000	0.0001	NA	NA
LMM												
β_0	-0.0019	0.0448	0.0447	0.954	0.0007	0.0452	0.0447	0.941	-0.0036	0.0466	0.0447	0.937
β_1	0.0016	0.0352	0.0360	0.958	0.0002	0.0384	0.0387	0.955	0.0007	0.0379	0.0389	0.953
β_2	0.0017	0.0365	0.0360	0.944	-0.0004	0.0371	0.0387	0.963	-0.0006	0.0464	0.0457	0.941
σ^2	0.0008	0.0365	NA	NA	0.0025	0.0371	NA	NA	0.0025	0.0464	NA	NA
ρ	-0.2687	0.0000	NA	NA	-0.2671	0.0001	NA	NA	-0.2653	0.0001	NA	NA
GEE												
β_0	-0.0019	0.0448	0.0447	0.951	0.0007	0.0452	0.0447	0.938	-0.0036	0.0466	0.0448	0.939
β_1	0.0016	0.0352	0.0360	0.957	0.0002	0.0384	0.0387	0.952	0.0007	0.0379	0.0389	0.952
β_2	0.0017	0.0365	0.0360	0.945	-0.0004	0.0371	0.0387	0.961	-0.0006	0.0464	0.0456	0.940
σ^2	-0.0011	0.0365	NA	NA	0.0002	0.0371	NA	NA	-0.0003	0.0464	NA	NA
ρ	-0.0001	0.0001	NA	NA	-0.0004	0.0001	NA	NA	-0.0007	0.0001	NA	NA
PROPOSED												
β_0	-0.0019	0.0448	0.0447	0.953	0.0007	0.0452	0.0448	0.944	-0.0036	0.0466	0.0448	0.936
β_1	0.0016	0.0352	0.0360	0.959	0.0002	0.0384	0.0387	0.954	0.0007	0.0379	0.0389	0.949
β_2	0.0017	0.0365	0.0360	0.944	-0.0004	0.0371	0.0387	0.963	-0.0006	0.0464	0.0457	0.940
σ^2	0.0015	0.0365	NA	NA	0.0034	0.0371	NA	NA	0.0032	0.0464	NA	NA
ρ	-0.0002	0.0000	NA	NA	-0.0004	0.0001	NA	NA	-0.0009	0.0001	NA	NA

Table 1.2. Estimates of three parameter missing analyses under MCAR with missing rates of (time1, time2, time3)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.7$, Correlation structure: AR (1)

Missing rate	(10%, 10%, 10%)				(25%, 25%, 25%)				(10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	0.0011	0.0516	0.0524	0.953	0.0014	0.0675	0.0686	0.955	0.0035	0.0764	0.0769	0.951
β_1	-0.0002	0.0411	0.0406	0.946	-0.0006	0.0531	0.0533	0.945	-0.0015	0.0606	0.0596	0.940
β_2	-0.0003	0.0531	0.0529	0.943	-0.0016	0.0699	0.0694	0.937	-0.0015	0.0788	0.0776	0.944
σ^2	0.0002	0.0531	NA	NA	-0.0083	0.0699	NA	NA	-0.0045	0.0788	NA	NA
ρ	-0.0002	0.0001	NA	NA	-0.0009	0.0003	NA	NA	-0.0005	0.0005	NA	NA
LOCF												
β_0	-0.0007	0.0455	0.0459	0.946	0.0007	0.0480	0.0480	0.946	0.0015	0.0458	0.0459	0.946
β_1	-0.0004	0.0349	0.0344	0.945	0.0002	0.0350	0.0342	0.946	-0.0006	0.0316	0.0314	0.957
β_2	-0.0004	0.0459	0.0452	0.942	-0.0009	0.0459	0.0451	0.949	-0.0005	0.0400	0.0395	0.952
σ^2	0.0025	0.0459	NA	NA	-0.0041	0.0459	NA	NA	-0.0007	0.0400	NA	NA
ρ	0.0192	0.0001	NA	NA	0.0533	0.0001	NA	NA	0.0984	0.0001	NA	NA
MI												
β_0	-0.0006	0.0457	0.0459	0.952	0.0029	0.0863	0.0445	0.743	0.0015	0.0542	0.0447	0.897
β_1	-0.0003	0.0385	0.0376	0.945	0.0021	0.0585	0.0489	0.898	0.0000	0.0538	0.0458	0.917
β_2	-0.0003	0.0491	0.0467	0.935	0.0009	0.0845	0.0535	0.811	0.0026	0.1499	0.0552	0.629
σ^2	0.0037	0.0491	NA	NA	-0.0147	0.0845	NA	NA	-0.0245	0.1499	NA	NA
ρ	-0.0362	0.0001	NA	NA	-0.2638	0.0003	NA	NA	-0.2990	0.0003	NA	NA
EM												
β_0	-0.0002	0.0554	0.0447	0.899	-0.0004	0.0514	0.0446	0.912	0.0015	0.0477	0.0447	0.926
β_1	-0.0006	0.0432	0.0415	0.948	0.0018	0.0459	0.0346	0.858	-0.0010	0.0433	0.0346	0.885
β_2	-0.0020	0.0578	0.0490	0.900	0.0005	0.0595	0.0450	0.862	-0.0015	0.0634	0.0450	0.840
σ^2	-0.0014	0.0578	NA	NA	-0.0058	0.0595	NA	NA	-0.0020	0.0634	NA	NA
ρ	-0.1053	0.0001	NA	NA	0.0001	0.0001	NA	NA	0.0007	0.0001	NA	NA
LMM												
β_o	-0.0006	0.0457	0.0459	0.952	0.0008	0.0489	0.0485	0.948	0.0014	0.0463	0.0460	0.943
β_1	-0.0003	0.0385	0.0376	0.945	0.0001	0.0437	0.0435	0.950	-0.0006	0.0401	0.0409	0.956
β_2	-0.0003	0.0491	0.0467	0.935	-0.0014	0.0552	0.0510	0.921	-0.0006	0.0593	0.0553	0.937
σ^2	0.0037	0.0491	NA	NA	-0.0030	0.0552	NA	NA	0.0011	0.0593	NA	NA
ρ	-0.0362	0.0001	NA	NA	-0.0614	0.0001	NA	NA	-0.0610	0.0001	NA	NA
GEE												
β_o	-0.0006	0.0467	0.0447	0.935	0.0009	0.0490	0.0492	0.954	0.0014	0.0464	0.0464	0.943
β_1	-0.0004	0.0408	0.0346	0.896	0.0001	0.0437	0.0433	0.951	-0.0006	0.0402	0.0404	0.952
β_2	-0.0005	0.0505	0.0451	0.923	-0.0014	0.0553	0.0543	0.941	-0.0005	0.0594	0.0589	0.956
σ^2	0.0016	0.0505	NA	NA	-0.0047	0.0553	NA	NA	-0.0009	0.0594	NA	NA
ρ	0.0004	0.0001	NA	NA	-0.0147	0.0002	NA	NA	-0.0128	0.0002	NA	NA
PROPOSED												
β_o	-0.0007	0.0457	0.0463	0.956	0.0006	0.0487	0.0488	0.947	0.0015	0.0460	0.0462	0.946
β_1	-0.0003	0.0385	0.0376	0.945	0.0002	0.0434	0.0428	0.946	-0.0006	0.0399	0.0402	0.959
β_2	-0.0003	0.0492	0.0485	0.945	-0.0012	0.0540	0.0530	0.950	-0.0007	0.0579	0.0575	0.953
σ^2	0.0019	0.0492	NA	NA	-0.0023	0.0540	NA	NA	0.0020	0.0579	NA	NA
ρ	-0.0055	0.0001	NA	NA	-0.0011	0.0001	NA	NA	-0.0008	0.0001	NA	NA

Table 1.3. Estimates of three parameter missing analyses under MCAR with missing rates of (time1, time2, time3)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: Exchangeable

Missing rate	(10%, 10%, 10%)				(25%, 25%, 25%)				(10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	0.0004	0.0492	0.0497	0.952	0.0018	0.0590	0.0596	0.948	0.0036	0.0736	0.0729	0.951
β_1	-0.0002	0.0644	0.0628	0.946	-0.0002	0.0767	0.0753	0.952	-0.0056	0.0891	0.0923	0.962
β_2	-0.0018	0.0620	0.0628	0.953	-0.0005	0.0786	0.0754	0.939	-0.0030	0.0901	0.0921	0.960
σ^2	-0.0025	0.0620	NA	NA	-0.0065	0.0786	NA	NA	-0.0079	0.0901	NA	NA
ρ	-0.0007	0.0002	NA	NA	-0.0006	0.0003	NA	NA	-0.0016	0.0007	NA	NA
LOCF												
β_0	0.0003	0.0439	0.0447	0.950	0.0026	0.0444	0.0447	0.953	0.0004	0.0454	0.0447	0.955
β_1	-0.0004	0.0550	0.0536	0.948	0.0002	0.0493	0.0490	0.947	-0.0013	0.0488	0.0490	0.955
β_2	-0.0017	0.0550	0.0563	0.952	-0.0018	0.0568	0.0548	0.943	0.0003	0.0527	0.0528	0.943
σ^2	-0.0031	0.0550	NA	NA	-0.0012	0.0568	NA	NA	-0.0025	0.0527	NA	NA
ρ	0.0552	0.0001	NA	NA	0.1497	0.0001	NA	NA	0.2330	0.0001	NA	NA

Table 1.3. Continue

MI												
β_0	0.0003	0.0439	0.0447	0.950	0.0026	0.0444	0.0447	0.953	0.0004	0.0454	0.0447	0.955
β_1	0.0001	0.0615	0.0572	0.944	-0.0004	0.0794	0.0582	0.865	-0.0028	0.0777	0.0582	0.875
β_2	-0.0022	0.0646	0.0573	0.919	-0.0016	0.0993	0.0583	0.784	0.0008	0.1773	0.0594	0.562
σ^2	-0.0037	0.0646	NA	NA	-0.0056	0.0993	NA	NA	-0.0238	0.1773	NA	NA
ρ	-0.0263	0.0001	NA	NA	-0.0618	0.0002	NA	NA	-0.0889	0.0003	NA	NA
EM												
β_0	0.0003	0.0439	0.0447	0.950	0.0026	0.0444	0.0447	0.953	0.0004	0.0454	0.0447	0.955
β_1	0.0002	0.0614	0.0565	0.936	-0.0017	0.0673	0.0566	0.907	-0.0015	0.0653	0.0565	0.904
β_2	-0.0013	0.0589	0.0566	0.935	-0.0030	0.0668	0.0565	0.901	-0.0008	0.0756	0.0565	0.855
σ^2	-0.0024	0.0589	NA	NA	-0.0036	0.0668	NA	NA	-0.0023	0.0756	NA	NA
ρ	-0.0005	0.0001	NA	NA	-0.0001	0.0002	NA	NA	0.0003	0.0002	NA	NA
LMM												
β_0	0.0003	0.0439	0.0447	0.949	0.0026	0.0444	0.0447	0.953	0.0004	0.0454	0.0447	0.957
β_1	-0.0002	0.0599	0.0584	0.945	-0.0004	0.0640	0.0619	0.945	-0.0017	0.0617	0.0619	0.953
β_2	-0.0017	0.0574	0.0584	0.951	-0.0020	0.0644	0.0619	0.946	0.0001	0.0696	0.0713	0.954
σ^2	-0.0005	0.0574	NA	NA	-0.0003	0.0644	NA	NA	-0.0006	0.0696	NA	NA
ρ	-0.4181	0.0001	NA	NA	-0.4146	0.0001	NA	NA	-0.4034	0.0002	NA	NA
GEE												
β_0	0.0003	0.0439	0.0447	0.950	0.0026	0.0444	0.0447	0.953	0.0004	0.0454	0.0447	0.955
β_1	-0.0002	0.0599	0.0583	0.945	-0.0004	0.0640	0.0618	0.945	-0.0017	0.0617	0.0619	0.953
β_2	-0.0017	0.0574	0.0584	0.948	-0.0020	0.0644	0.0618	0.942	0.0001	0.0696	0.0711	0.954
σ^2	-0.0026	0.0574	NA	NA	-0.0027	0.0644	NA	NA	-0.0033	0.0696	NA	NA
ρ	-0.0006	0.0002	NA	NA	-0.0006	0.0002	NA	NA	-0.0004	0.0003	NA	NA
PROPOSED												
β_0	0.0003	0.0439	0.0447	0.949	0.0026	0.0444	0.0447	0.952	0.0004	0.0454	0.0447	0.956
β_1	-0.0002	0.0599	0.0584	0.944	-0.0004	0.0640	0.0619	0.945	-0.0017	0.0617	0.0619	0.953
β_2	-0.0017	0.0574	0.0584	0.951	-0.0020	0.0644	0.0619	0.944	0.0001	0.0695	0.0713	0.948
σ^2	-0.0003	0.0574	NA	NA	0.0002	0.0644	NA	NA	0.0000	0.0695	NA	NA
ρ	-0.0007	0.0001	NA	NA	-0.0006	0.0001	NA	NA	-0.0016	0.0002	NA	NA

Table 1.4. Estimates of three parameter missing analyses under MCAR with missing rates of (time1, time2, time3)

Gold standard: β_0 (Baseline) = 1, β_1 ($time_{(2-1)}$) = 0, β_2 ($time_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: AR (1)

Missing rate	(10%, 10%, 10%)				(25%, 25%, 25%)				(10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	0.0004	0.0511	0.0524	0.960	0.0022	0.0711	0.0687	0.939	0.0049	0.0790	0.0768	0.942
β_1	-0.0010	0.0685	0.0662	0.945	-0.0049	0.0890	0.0869	0.945	-0.0005	0.0995	0.0973	0.938
β_2	-0.0023	0.0737	0.0725	0.950	-0.0070	0.0983	0.0951	0.932	-0.0026	0.1052	0.1066	0.951
σ^2	-0.0014	0.0737	NA	NA	-0.0104	0.0983	NA	NA	-0.0039	0.1052	NA	NA
ρ	0.0011	0.0003	NA	NA	0.0010	0.0006	NA	NA	0.0006	0.0009	NA	NA
LOCF												
β_0	0.0000	0.0469	0.0470	0.954	0.0015	0.0502	0.0507	0.953	0.0004	0.0473	0.0468	0.939
β_1	-0.0003	0.0579	0.0556	0.936	-0.0040	0.0559	0.0547	0.944	0.0013	0.0520	0.0510	0.948
β_2	-0.0020	0.0648	0.0628	0.938	-0.0045	0.0632	0.0637	0.951	0.0004	0.0579	0.0576	0.941
σ^2	-0.0015	0.0648	NA	NA	-0.0037	0.0632	NA	NA	-0.0017	0.0579	NA	NA
ρ	0.0713	0.0002	NA	NA	0.1822	0.0002	NA	NA	0.2870	0.0002	NA	NA
MI												
β_0	-0.0023	0.0565	0.0447	0.887	0.0003	0.0885	0.0444	0.703	0.0006	0.0565	0.0446	0.889
β_1	0.0012	0.0636	0.0578	0.929	-0.0035	0.0766	0.0592	0.889	0.0023	0.0686	0.0586	0.916
β_2	0.0002	0.0742	0.0621	0.901	-0.0033	0.1022	0.0621	0.811	0.0034	0.1579	0.0620	0.640
σ^2	-0.0036	0.0742	NA	NA	-0.0202	0.1022	NA	NA	-0.0293	0.1579	NA	NA
ρ	-0.0356	0.0002	NA	NA	-0.0831	0.0003	NA	NA	-0.0902	0.0003	NA	NA
EM												
β_0	0.0003	0.0494	0.0447	0.930	0.0016	0.0549	0.0446	0.895	0.0006	0.0492	0.0446	0.920
β_1	0.0002	0.0665	0.0565	0.903	-0.0039	0.0745	0.0565	0.868	0.0013	0.0674	0.0565	0.899
β_2	-0.0023	0.0705	0.0619	0.915	-0.0049	0.0775	0.0618	0.885	0.0006	0.0830	0.0619	0.856
σ^2	-0.0016	0.0705	NA	NA	-0.0060	0.0775	NA	NA	-0.0027	0.0830	NA	NA
ρ	0.0008	0.0002	NA	NA	0.0004	0.0002	NA	NA	0.0005	0.0003	NA	NA
LMM												
β_0	0.0000	0.0471	0.0470	0.953	0.0013	0.0508	0.0513	0.957	0.0003	0.0474	0.0470	0.939
β_1	-0.0004	0.0630	0.0603	0.935	-0.0036	0.0689	0.0673	0.941	0.0019	0.0643	0.0640	0.948
β_2	-0.0014	0.0674	0.0645	0.944	-0.0047	0.0702	0.0702	0.948	0.0016	0.0755	0.0760	0.946
σ^2	0.0010	0.0674	NA	NA	-0.0031	0.0702	NA	NA	0.0000	0.0755	NA	NA
ρ	-0.0255	0.0001	NA	NA	-0.0409	0.0002	NA	NA	-0.0404	0.0002	NA	NA

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Table 1.4. Continue

GEE												
β_0	-0.0001	0.0471	0.0471	0.954	0.0013	0.0508	0.0514	0.956	0.0003	0.0474	0.0470	0.941
β_1	-0.0004	0.0630	0.0602	0.934	-0.0037	0.0690	0.0668	0.941	0.0018	0.0643	0.0636	0.947
β_2	-0.0015	0.0674	0.0654	0.946	-0.0047	0.0702	0.0717	0.951	0.0016	0.0755	0.0775	0.949
σ^2	-0.0012	0.0674	NA	NA	-0.0056	0.0702	NA	NA	-0.0027	0.0755	NA	NA
ρ	-0.0063	0.0002	NA	NA	-0.0165	0.0002	NA	NA	-0.0139	0.0003	NA	NA
PROPOSED												
β_0	0.0000	0.0470	0.0471	0.952	0.0014	0.0506	0.0514	0.956	0.0003	0.0474	0.0471	0.943
β_1	-0.0004	0.0629	0.0602	0.933	-0.0039	0.0688	0.0668	0.941	0.0019	0.0644	0.0636	0.947
β_2	-0.0015	0.0673	0.0653	0.947	-0.0051	0.0702	0.0716	0.956	0.0017	0.0753	0.0774	0.950
σ^2	0.0013	0.0673	NA	NA	-0.0022	0.0702	NA	NA	0.0013	0.0753	NA	NA
ρ	0.0008	0.0001	NA	NA	0.0004	0.0002	NA	NA	0.0006	0.0002	NA	NA

Table 2.1. Estimates from missing analyses in three parameter model under MAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI												
β_0	-0.0017	0.0436	0.0447	0.948	0.0005	0.0458	0.0448	0.943	-0.0001	0.0448	0.0447	0.943
β_1	-0.1049	0.0408	0.0406	0.271	-0.1911	0.0492	0.0438	0.009	-0.4179	0.0824	0.0491	0.001
β_2	-0.1061	0.0411	0.0388	0.216	-0.1933	0.0495	0.0413	0.006	-0.4196	0.0875	0.0461	0.000
σ^2	-0.1232	0.0411	NA	NA	-0.1849	0.0495	NA	NA	-0.3141	0.0875	NA	NA
ρ	-0.1271	0.0001	NA	NA	-0.2012	0.0001	NA	NA	-0.3549	0.0003	NA	NA
EM												
β_0	-0.0017	0.0436	0.0447	0.948	0.0005	0.0458	0.0448	0.943	-0.0001	0.0448	0.0447	0.943
β_1	-0.0028	0.0362	0.0347	0.938	-0.0090	0.0381	0.0348	0.933	-0.0213	0.0468	0.0350	0.825
β_2	-0.0084	0.0367	0.0348	0.935	-0.0239	0.0378	0.0351	0.879	-0.0769	0.0452	0.0356	0.446
σ^2	-0.0186	0.0367	NA	NA	-0.0337	0.0378	NA	NA	-0.0892	0.0452	NA	NA
ρ	-0.0052	0.0001	NA	NA	-0.0098	0.0001	NA	NA	-0.0250	0.0001	NA	NA
LMM												
β_0	-0.0017	0.0436	0.0446	0.946	0.0005	0.0458	0.0444	0.938	-0.0001	0.0448	0.0440	0.943
β_1	-0.0029	0.0355	0.0354	0.949	-0.0093	0.0362	0.0363	0.947	-0.0252	0.0397	0.0394	0.898
β_2	-0.0069	0.0356	0.0354	0.944	-0.0194	0.0360	0.0363	0.918	-0.0540	0.0409	0.0394	0.728
σ^2	-0.0146	0.0356	NA	NA	-0.0265	0.0360	NA	NA	-0.0665	0.0409	NA	NA
ρ	-0.2729	0.0000	NA	NA	-0.2767	0.0000	NA	NA	-0.2925	0.0001	NA	NA
GEE												
β_0	-0.0017	0.0436	0.0447	0.948	0.0005	0.0458	0.0448	0.943	-0.0001	0.0448	0.0447	0.943
β_1	-0.0124	0.0358	0.0357	0.933	-0.0334	0.0368	0.0368	0.856	-0.0836	0.0414	0.0403	0.460
β_2	-0.0160	0.0358	0.0356	0.922	-0.0424	0.0366	0.0367	0.799	-0.1089	0.0410	0.0400	0.214
σ^2	-0.1168	0.0358	NA	NA	-0.1637	0.0366	NA	NA	-0.2159	0.0410	NA	NA
ρ	-0.0859	0.0001	NA	NA	-0.1194	0.0001	NA	NA	-0.1391	0.0001	NA	NA
PROPOSED												
β_0	-0.0017	0.0436	0.0428	0.935	0.0005	0.0458	0.0420	0.917	-0.0001	0.0448	0.0408	0.918
β_1	-0.0086	0.0356	0.0367	0.953	-0.0256	0.0365	0.0383	0.916	-0.0896	0.0414	0.0432	0.458
β_2	-0.0124	0.0357	0.0367	0.940	-0.0350	0.0364	0.0383	0.882	-0.1145	0.0422	0.0432	0.233
σ^2	-0.1677	0.0357	NA	NA	-0.2348	0.0364	NA	NA	-0.3350	0.0422	NA	NA
ρ	-0.0538	0.0000	NA	NA	-0.0843	0.0001	NA	NA	-0.1508	0.0001	NA	NA

Table 2.2. Estimates of three parameter missing analyses under MAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: AR (1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	0.0014	0.0433	0.0447	0.966	0.0012	0.0443	0.0447	0.95	0.0008	0.0429	0.0447	0.960
β_1	-0.1091	0.0393	0.0407	0.208	-0.1907	0.0500	0.0437	0.013	-0.4170	0.0851	0.0490	0.000
β_2	-0.1099	0.0477	0.0480	0.385	-0.1946	0.0576	0.0496	0.039	-0.4205	0.0957	0.0524	0.002
σ^2	-0.1262	0.0477	NA	NA	-0.1925	0.0576	NA	NA	-0.3122	0.0957	NA	NA
ρ	-0.1104	0.0001	NA	NA	-0.1829	0.0002	NA	NA	-0.3438	0.0003	NA	NA
EM												
β_0	0.0014	0.0433	0.0447	0.966	0.0012	0.0443	0.0447	0.95	0.0008	0.0429	0.0447	0.960
β_1	-0.0090	0.0347	0.0348	0.937	-0.0160	0.0391	0.0350	0.905	-0.0459	0.0483	0.0353	0.676
β_2	-0.0176	0.0454	0.0456	0.931	-0.0379	0.0493	0.0460	0.848	-0.1322	0.0595	0.0473	0.254
σ^2	-0.0289	0.0454	NA	NA	-0.0562	0.0493	NA	NA	-0.1243	0.0595	NA	NA
ρ	-0.0098	0.0001	NA	NA	-0.0201	0.0001	NA	NA	-0.0502	0.0001	NA	NA

Table 2.2. Continue

LMM												
β_0	0.0121	0.0439	0.0442	0.942	0.0180	0.0450	0.0438	0.926	0.0251	0.0435	0.0432	0.918
β_1	-0.0280	0.0348	0.0358	0.882	-0.0472	0.0383	0.0370	0.745	-0.1013	0.0428	0.0407	0.315
β_2	-0.0556	0.0467	0.0458	0.769	-0.0983	0.0498	0.0467	0.456	-0.2162	0.0570	0.0500	0.018
σ^2	-0.0430	0.0467	NA	NA	-0.0725	0.0498	NA	NA	-0.1215	0.0570	NA	NA
ρ	-0.0410	0.0001	NA	NA	-0.0511	0.0001	NA	NA	-0.0710	0.0001	NA	NA
GEE												
β_0	0.0098	0.0437	0.0452	0.958	0.0128	0.0447	0.0452	0.941	0.0157	0.0434	0.0451	0.953
β_1	-0.0321	0.0349	0.0361	0.865	-0.0588	0.0386	0.0374	0.657	-0.1373	0.0435	0.0409	0.096
β_2	-0.0639	0.0466	0.0479	0.74	-0.1180	0.0495	0.0496	0.331	-0.2657	0.0554	0.0534	0.001
σ^2	-0.1215	0.0466	NA	NA	-0.1775	0.0495	NA	NA	-0.2411	0.0554	NA	NA
ρ	-0.0801	0.0001	NA	NA	-0.1185	0.0001	NA	NA	-0.1662	0.0001	NA	NA
PROPOSED												
β_0	0.0014	0.0433	0.0432	0.959	0.0012	0.0443	0.0425	0.939	0.0008	0.0429	0.0415	0.943
β_1	-0.0124	0.0343	0.0365	0.943	-0.0263	0.0379	0.0379	0.904	-0.0873	0.0423	0.0425	0.475
β_2	-0.0229	0.0450	0.0468	0.932	-0.0516	0.0481	0.0482	0.822	-0.1647	0.0541	0.0525	0.127
σ^2	-0.1378	0.0450	NA	NA	-0.1974	0.0481	NA	NA	-0.2758	0.0541	NA	NA
ρ	-0.0450	0.0001	NA	NA	-0.0712	0.0001	NA	NA	-0.1245	0.0001	NA	NA

Table 2.3. Estimates of three parameter missing analyses under MAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2\text{-}1)}$) = 0, β_2 (time$_{(3\text{-}1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	0.0014	0.0436	0.0446	0.954	-0.0005	0.0438	0.0447	0.954	-0.0015	0.0448	0.0447	0.959
β_1	-0.0328	0.0564	0.0579	0.917	-0.0537	0.0603	0.0588	0.849	-0.1083	0.0780	0.0601	0.545
β_2	-0.0321	0.0575	0.0570	0.910	-0.0536	0.0647	0.0574	0.827	-0.1174	0.0984	0.0584	0.442
σ^2	-0.0161	0.0575	NA	NA	-0.0148	0.0647	NA	NA	-0.0282	0.0984	NA	NA
ρ	-0.0385	0.0001	NA	NA	-0.0585	0.0002	NA	NA	-0.0985	0.0002	NA	NA
EM												
β_0	0.0014	0.0436	0.0446	0.954	-0.0005	0.0438	0.0447	0.954	-0.0015	0.0448	0.0447	0.959
β_1	-0.0042	0.0575	0.0566	0.943	-0.0034	0.0597	0.0567	0.947	-0.0030	0.0684	0.0567	0.900
β_2	-0.0037	0.0571	0.0566	0.952	-0.0066	0.0602	0.0568	0.941	-0.0336	0.0665	0.0572	0.862
σ^2	-0.0073	0.0571	NA	NA	-0.0044	0.0602	NA	NA	-0.0128	0.0665	NA	NA
ρ	-0.0023	0.0002	NA	NA	-0.0045	0.0002	NA	NA	-0.0157	0.0002	NA	NA
LMM												
β_0	0.0014	0.0436	0.0447	0.955	-0.0005	0.0438	0.0447	0.957	-0.0015	0.0448	0.0446	0.961
β_1	-0.0040	0.0561	0.0574	0.948	-0.0040	0.0582	0.0585	0.959	-0.0109	0.0614	0.0623	0.956
β_2	-0.0042	0.0559	0.0574	0.962	-0.0071	0.0586	0.0585	0.946	-0.0338	0.0605	0.0623	0.914
σ^2	-0.0050	0.0559	NA	NA	-0.0022	0.0586	NA	NA	-0.0108	0.0605	NA	NA
ρ	-0.4187	0.0001	NA	NA	-0.4187	0.0001	NA	NA	-0.4069	0.0002	NA	NA
GEE												
β_0	0.0014	0.0436	0.0446	0.954	-0.0005	0.0438	0.0447	0.954	-0.0015	0.0448	0.0447	0.959
β_1	-0.0072	0.0562	0.0575	0.947	-0.0113	0.0583	0.0587	0.958	-0.0222	0.0614	0.0624	0.951
β_2	-0.0072	0.0560	0.0574	0.959	-0.0136	0.0585	0.0586	0.944	-0.0428	0.0600	0.0624	0.902
σ^2	-0.0152	0.0560	NA	NA	-0.0144	0.0585	NA	NA	-0.0198	0.0600	NA	NA
ρ	-0.0265	0.0001	NA	NA	-0.0356	0.0002	NA	NA	-0.0396	0.0002	NA	NA
PROPOSED												
β_0	0.0014	0.0436	0.0445	0.953	-0.0005	0.0438	0.0445	0.955	-0.0015	0.0448	0.0444	0.958
β_1	-0.0074	0.0562	0.0581	0.950	-0.0129	0.0583	0.0595	0.958	-0.0399	0.0626	0.0638	0.916
β_2	-0.0074	0.0560	0.0581	0.960	-0.0151	0.0586	0.0595	0.945	-0.0568	0.0611	0.0638	0.869
σ^2	-0.0204	0.0560	NA	NA	-0.0222	0.0586	NA	NA	-0.0324	0.0611	NA	NA
ρ	-0.0280	0.0001	NA	NA	-0.0419	0.0001	NA	NA	-0.0717	0.0002	NA	NA

Table 2.4. Estimates of three parameter missing analyses under MAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2\text{-}1)}$) = 0, β_2 (time$_{(3\text{-}1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: AR (1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI												
β_0	-0.0047	0.0455	0.0447	0.941	0.0017	0.0452	0.0447	0.949	0.0008	0.0455	0.0447	0.940
β_1	-0.0275	0.0599	0.0580	0.914	-0.0551	0.0601	0.0588	0.853	-0.1163	0.0791	0.0602	0.517
β_2	-0.0246	0.0653	0.0622	0.922	-0.0585	0.0708	0.0622	0.816	-0.1242	0.1067	0.0623	0.481
σ^2	-0.0093	0.0653	NA	NA	-0.0176	0.0708	NA	NA	-0.0314	0.1067	NA	NA
ρ	-0.0467	0.0002	NA	NA	-0.0735	0.0002	NA	NA	-0.1231	0.0002	NA	NA

Table 2.4. Continue

EM												
β_0	-0.0047	0.0455	0.0447	0.941	0.0017	0.0452	0.0447	0.949	0.0008	0.0455	0.0447	0.940
β_1	0.0016	0.0608	0.0566	0.924	-0.0030	0.0597	0.0566	0.932	-0.0058	0.0696	0.0567	0.893
β_2	0.0034	0.0642	0.0622	0.930	-0.0132	0.0672	0.0622	0.924	-0.0436	0.0749	0.0628	0.837
σ^2	0.0004	0.0642	NA	NA	-0.0037	0.0672	NA	NA	-0.0101	0.0749	NA	NA
ρ	-0.0016	0.0002	NA	NA	-0.0055	0.0002	NA	NA	-0.0156	0.0002	NA	NA
LMM												
β_o	-0.0020	0.0456	0.0447	0.942	0.0054	0.0453	0.0446	0.948	0.0047	0.0455	0.0445	0.935
β_1	-0.0070	0.0601	0.0580	0.932	-0.0196	0.0583	0.0594	0.944	-0.0496	0.0629	0.0637	0.875
β_2	-0.0212	0.0645	0.0623	0.931	-0.0528	0.0646	0.0631	0.867	-0.1100	0.0715	0.0663	0.592
σ^2	-0.0034	0.0645	NA	NA	-0.0090	0.0646	NA	NA	-0.0194	0.0715	NA	NA
ρ	-0.0350	0.0001	NA	NA	-0.0506	0.0001	NA	NA	-0.0751	0.0002	NA	NA
GEE												
β_0	-0.0023	0.0456	0.0449	0.942	0.0048	0.0454	0.0448	0.950	0.0037	0.0455	0.0447	0.939
β_1	-0.0086	0.0601	0.0578	0.934	-0.0235	0.0583	0.0590	0.942	-0.0594	0.0630	0.0631	0.835
β_2	-0.0233	0.0644	0.0638	0.933	-0.0568	0.0646	0.0648	0.864	-0.1157	0.0710	0.0681	0.586
σ^2	-0.0094	0.0644	NA	NA	-0.0159	0.0646	NA	NA	-0.0257	0.0710	NA	NA
ρ	-0.0366	0.0002	NA	NA	-0.0560	0.0002	NA	NA	-0.0865	0.0002	NA	NA
PROPOSED												
β_0	-0.0047	0.0455	0.0446	0.941	0.0017	0.0452	0.0445	0.949	0.0008	0.0455	0.0444	0.939
β_1	-0.0018	0.0599	0.0581	0.936	-0.0123	0.0582	0.0594	0.947	-0.0390	0.0632	0.0635	0.907
β_2	-0.0001	0.0635	0.0629	0.943	-0.0209	0.0641	0.0638	0.939	-0.0622	0.0705	0.0672	0.835
σ^2	-0.0090	0.0635	NA	NA	-0.0154	0.0641	NA	NA	-0.0246	0.0705	NA	NA
ρ	-0.0236	0.0001	NA	NA	-0.0369	0.0001	NA	NA	-0.0605	0.0001	NA	NA

Table 3.1. Estimates of three parameter missing analyses under MNAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI												
β_0	0.0019	0.0464	0.0447	0.941	-0.0004	0.0452	0.0447	0.942	0.0003	0.0432	0.0447	0.962
β_1	-0.1538	0.0402	0.0383	0.023	-0.2740	0.0459	0.0404	0.000	-0.5954	0.0666	0.0438	0.000
β_2	-0.1548	0.0413	0.0383	0.028	-0.2767	0.0487	0.0404	0.000	-0.5931	0.0778	0.0438	0.000
σ^2	-0.2553	0.0413	NA	NA	-0.3832	0.0487	NA	NA	-0.6313	0.0778	NA	NA
ρ	-0.1102	0.0001	NA	NA	-0.1799	0.0001	NA	NA	-0.3259	0.0002	NA	NA
EM												
β_0	0.0019	0.0464	0.0447	0.941	-0.0004	0.0452	0.0447	0.942	0.0003	0.0432	0.0447	0.962
β_1	-0.0830	0.0362	0.0345	0.337	-0.1558	0.0366	0.0345	0.011	-0.3871	0.0395	0.0351	0.000
β_2	-0.0816	0.0363	0.0344	0.351	-0.1556	0.0370	0.0345	0.009	-0.3865	0.0408	0.0351	0.000
σ^2	-0.2059	0.0363	NA	NA	-0.3205	0.0370	NA	NA	-0.5551	0.0408	NA	NA
ρ	-0.0241	0.0001	NA	NA	-0.0420	0.0001	NA	NA	-0.0933	0.0001	NA	NA
LMM												
β_0	0.0019	0.0464	0.0425	0.929	-0.0004	0.0452	0.0414	0.917	0.0003	0.0432	0.0396	0.924
β_1	-0.0783	0.0356	0.0347	0.405	-0.1414	0.0355	0.0352	0.021	-0.3268	0.0371	0.0374	0.000
β_2	-0.0772	0.0356	0.0347	0.401	-0.1417	0.0359	0.0352	0.027	-0.3260	0.0376	0.0374	0.000
σ^2	-0.1918	0.0356	NA	NA	-0.2843	0.0359	NA	NA	-0.4296	0.0376	NA	NA
ρ	-0.2945	0.0000	NA	NA	-0.3151	0.0000	NA	NA	-0.3687	0.0001	NA	NA
GEE												
β_0	0.0019	0.0464	0.0447	0.941	-0.0004	0.0452	0.0447	0.942	0.0003	0.0432	0.0447	0.962
β_1	-0.0826	0.0357	0.0351	0.374	-0.1529	0.0356	0.0357	0.012	-0.3630	0.0369	0.0378	0.000
β_2	-0.0816	0.0358	0.0350	0.360	-0.1531	0.0362	0.0357	0.009	-0.3624	0.0374	0.0378	0.000
σ^2	-0.2491	0.0358	NA	NA	-0.3609	0.0362	NA	NA	-0.5269	0.0374	NA	NA
ρ	-0.0694	0.0001	NA	NA	-0.1023	0.0001	NA	NA	-0.1531	0.0001	NA	NA
PROPOSED												
β_0	0.0019	0.0464	0.0414	0.922	-0.0004	0.0452	0.0399	0.906	0.0003	0.0432	0.0376	0.908
β_1	-0.0812	0.0357	0.0355	0.388	-0.1499	0.0356	0.0364	0.016	-0.3621	0.0373	0.0398	0.000
β_2	-0.0801	0.0357	0.0355	0.382	-0.1501	0.0362	0.0364	0.018	-0.3614	0.0377	0.0398	0.000
σ^2	-0.2844	0.0357	NA	NA	-0.4052	0.0362	NA	NA	-0.5863	0.0377	NA	NA
ρ	-0.0538	0.0000	NA	NA	-0.0841	0.0001	NA	NA	-0.1505	0.0001	NA	NA

Table 3.2. Estimates of three parameter missing analyses under MNAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.7$, Correlation structure: AR (1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI												
β_0	-0.0008	0.0464	0.0446	0.945	0.0010	0.0451	0.0447	0.943	-0.0017	0.0433	0.0446	0.961
β_1	-0.1508	0.0399	0.0382	0.023	-0.2733	0.0469	0.0403	0.000	-0.5968	0.0704	0.0439	0.000
β_2	-0.1534	0.0480	0.0459	0.080	-0.2749	0.0541	0.0465	0.001	-0.5968	0.0805	0.0477	0.000
σ^2	-0.2582	0.0480	NA	NA	-0.3836	0.0541	NA	NA	-0.6288	0.0805	NA	NA
ρ	-0.0886	0.0001	NA	NA	-0.1492	0.0001	NA	NA	-0.3011	0.0002	NA	NA
EM												
β_0	-0.0008	0.0464	0.0446	0.945	0.0010	0.0451	0.0447	0.943	-0.0017	0.0433	0.0446	0.961
β_1	-0.0747	0.0348	0.0343	0.401	-0.1488	0.0356	0.0343	0.010	-0.3722	0.0391	0.0348	0.000
β_2	-0.1041	0.0444	0.0443	0.346	-0.2000	0.0458	0.0440	0.007	-0.4776	0.0483	0.0438	0.000
σ^2	-0.2209	0.0444	NA	NA	-0.3407	0.0458	NA	NA	-0.5831	0.0483	NA	NA
ρ	-0.0245	0.0001	NA	NA	-0.0440	0.0001	NA	NA	-0.0974	0.0001	NA	NA
LMM												
β_0	-0.0026	0.0464	0.0423	0.940	-0.0003	0.0452	0.0410	0.921	-0.0012	0.0434	0.0390	0.928
β_1	-0.0758	0.0345	0.0346	0.398	-0.1459	0.0349	0.0351	0.015	-0.3365	0.0371	0.0372	0.000
β_2	-0.1113	0.0441	0.0441	0.278	-0.2097	0.0451	0.0441	0.003	-0.4760	0.0471	0.0454	0.000
σ^2	-0.2099	0.0441	NA	NA	-0.3087	0.0451	NA	NA	-0.4650	0.0471	NA	NA
ρ	-0.0492	0.0001	NA	NA	-0.0644	0.0001	NA	NA	-0.0946	0.0001	NA	NA
GEE												
β_0	-0.0035	0.0463	0.0446	0.952	-0.0021	0.0451	0.0447	0.949	-0.0053	0.0433	0.0447	0.960
β_1	-0.0779	0.0346	0.0349	0.384	-0.1517	0.0350	0.0355	0.011	-0.3600	0.0372	0.0374	0.000
β_2	-0.1150	0.0442	0.0450	0.265	-0.2185	0.0452	0.0455	0.002	-0.5033	0.0469	0.0469	0.000
σ^2	-0.2535	0.0442	NA	NA	-0.3669	0.0452	NA	NA	-0.5444	0.0469	NA	NA
ρ	-0.0611	0.0001	NA	NA	-0.0921	0.0001	NA	NA	-0.1537	0.0001	NA	NA
PROPOSED												
β_0	-0.0008	0.0464	0.0416	0.930	0.0010	0.0451	0.0402	0.922	-0.0017	0.0433	0.0379	0.917
β_1	-0.0726	0.0344	0.0352	0.460	-0.1415	0.0347	0.0359	0.021	-0.3400	0.0374	0.0388	0.000
β_2	-0.1036	0.0439	0.0452	0.362	-0.1972	0.0447	0.0456	0.008	-0.4651	0.0468	0.0479	0.000
σ^2	-0.2654	0.0439	NA	NA	-0.3805	0.0447	NA	NA	-0.5631	0.0468	NA	NA
ρ	-0.0455	0.0001	NA	NA	-0.0703	0.0001	NA	NA	-0.1246	0.0001	NA	NA

Table 3.3. Estimates of three parameter missing analyses under MNAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI												
β_0	-0.0008	0.0433	0.0448	0.955	-0.0004	0.0450	0.0447	0.946	-0.0017	0.0440	0.0447	0.957
β_1	-0.1525	0.0546	0.0549	0.202	-0.2767	0.0604	0.0539	0.000	-0.5967	0.0811	0.0523	0.000
β_2	-0.1562	0.0581	0.0548	0.196	-0.2785	0.0629	0.0539	0.001	-0.5997	0.0893	0.0523	0.000
σ^2	-0.2535	0.0581	NA	NA	-0.3858	0.0629	NA	NA	-0.6272	0.0893	NA	NA
ρ	-0.0361	0.0001	NA	NA	-0.0588	0.0001	NA	NA	-0.1033	0.0002	NA	NA
EM												
β_0	-0.0008	0.0433	0.0448	0.955	-0.0004	0.0450	0.0447	0.946	-0.0017	0.0440	0.0447	0.957
β_1	-0.1469	0.0545	0.0545	0.224	-0.2668	0.0564	0.0533	0.002	-0.5811	0.0566	0.0513	0.000
β_2	-0.1483	0.0566	0.0544	0.229	-0.2645	0.0561	0.0533	0.000	-0.5810	0.0569	0.0512	0.000
σ^2	-0.2545	0.0566	NA	NA	-0.3867	0.0561	NA	NA	-0.6220	0.0569	NA	NA
ρ	-0.0235	0.0001	NA	NA	-0.0374	0.0001	NA	NA	-0.0645	0.0001	NA	NA
LMM												
β_0	-0.0008	0.0433	0.0419	0.940	-0.0004	0.0450	0.0404	0.919	-0.0017	0.0440	0.0380	0.919
β_1	-0.1458	0.0537	0.0545	0.221	-0.2650	0.0553	0.0538	0.001	-0.5766	0.0547	0.0543	0.000
β_2	-0.1475	0.0560	0.0545	0.237	-0.2638	0.0547	0.0538	0.000	-0.5769	0.0543	0.0543	0.000
σ^2	-0.2478	0.0560	NA	NA	-0.3708	0.0547	NA	NA	-0.5572	0.0543	NA	NA
ρ	-0.4346	0.0001	NA	NA	-0.4434	0.0001	NA	NA	-0.4413	0.0001	NA	NA

Table 3.3. Continue

GEE												
β_0	-0.0008	0.0433	0.0448	0.955	-0.0004	0.0450	0.0447	0.946	-0.0017	0.0440	0.0447	0.957
β_1	-0.1459	0.0537	0.0552	0.230	-0.2652	0.0553	0.0547	0.001	-0.5776	0.0547	0.0546	0.000
β_2	-0.1476	0.0560	0.0552	0.240	-0.2640	0.0547	0.0547	0.000	-0.5778	0.0543	0.0545	0.000
σ^2	-0.2502	0.0560	NA	NA	-0.3732	0.0547	NA	NA	-0.5601	0.0543	NA	NA
ρ	-0.0250	0.0001	NA	NA	-0.0388	0.0001	NA	NA	-0.0660	0.0001	NA	NA
PROPOSED												
β_0	-0.0008	0.0433	0.0418	0.940	-0.0004	0.0450	0.0403	0.918	-0.0017	0.0440	0.0379	0.919
β_1	-0.1460	0.0537	0.0546	0.221	-0.2654	0.0553	0.0540	0.001	-0.5782	0.0547	0.0545	0.000
β_2	-0.1477	0.0560	0.0546	0.237	-0.2643	0.0547	0.0540	0.000	-0.5784	0.0544	0.0545	0.000
σ^2	-0.2503	0.0560	NA	NA	-0.3736	0.0547	NA	NA	-0.5601	0.0544	NA	NA
ρ	-0.0276	0.0001	NA	NA	-0.0425	0.0001	NA	NA	-0.0699	0.0001	NA	NA

Table 3.4. Estimates of three parameter missing analyses under MNAR

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: AR (1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI												
β_0	-0.0004	0.0448	0.0447	0.943	0.0023	0.0442	0.0447	0.955	-0.0013	0.0455	0.0447	0.956
β_1	-0.1514	0.0560	0.0549	0.207	-0.2753	0.0572	0.0539	0.001	-0.5956	0.0812	0.0523	0.000
β_2	-0.1528	0.0612	0.0592	0.275	-0.2789	0.0642	0.0576	0.004	-0.6006	0.0938	0.0547	0.000
σ^2	-0.2529	0.0612	NA	NA	-0.3863	0.0642	NA	NA	-0.6273	0.0938	NA	NA
ρ	-0.0415	0.0002	NA	NA	-0.0643	0.0002	NA	NA	-0.1110	0.0002	NA	NA
EM												
β_0	-0.0004	0.0448	0.0447	0.943	0.0023	0.0442	0.0447	0.955	-0.0013	0.0455	0.0447	0.956
β_1	-0.1432	0.0556	0.0545	0.252	-0.2656	0.0532	0.0533	0.000	-0.5791	0.0567	0.0512	0.000
β_2	-0.1492	0.0617	0.0591	0.285	-0.2735	0.0586	0.0575	0.003	-0.5945	0.0596	0.0546	0.000
σ^2	-0.2549	0.0617	NA	NA	-0.3864	0.0586	NA	NA	-0.6234	0.0596	NA	NA
ρ	-0.0284	0.0002	NA	NA	-0.0435	0.0002	NA	NA	-0.0727	0.0002	NA	NA
LMM												
β_0	-0.0026	0.0448	0.0418	0.932	-0.0005	0.0442	0.0404	0.927	-0.0038	0.0456	0.0380	0.895
β_1	-0.1423	0.0550	0.0545	0.257	-0.2629	0.0524	0.0539	0.001	-0.5748	0.0542	0.0544	0.000
β_2	-0.1497	0.0607	0.0585	0.270	-0.2754	0.0579	0.0572	0.001	-0.5981	0.0574	0.0565	0.000
σ^2	-0.2477	0.0607	NA	NA	-0.3693	0.0579	NA	NA	-0.5577	0.0574	NA	NA
ρ	-0.0381	0.0001	NA	NA	-0.0546	0.0001	NA	NA	-0.0866	0.0001	NA	NA
GEE												
β_0	-0.0027	0.0448	0.0447	0.946	-0.0007	0.0443	0.0447	0.957	-0.0040	0.0455	0.0447	0.951
β_1	-0.1424	0.0550	0.0552	0.263	-0.2632	0.0524	0.0547	0.001	-0.5764	0.0540	0.0544	0.000
β_2	-0.1499	0.0607	0.0598	0.286	-0.2758	0.0580	0.0588	0.001	-0.5992	0.0575	0.0577	0.000
σ^2	-0.2508	0.0607	NA	NA	-0.3725	0.0580	NA	NA	-0.5611	0.0575	NA	NA
ρ	-0.0310	0.0002	NA	NA	-0.0473	0.0002	NA	NA	-0.0821	0.0002	NA	NA
PROPOSED												
β_0	-0.0004	0.0448	0.0419	0.931	0.0023	0.0442	0.0404	0.928	-0.0013	0.0455	0.0380	0.897
β_1	-0.1435	0.0550	0.0545	0.249	-0.2642	0.0524	0.0539	0.001	-0.5748	0.0542	0.0542	0.000
β_2	-0.1487	0.0608	0.0590	0.277	-0.2731	0.0578	0.0579	0.002	-0.5933	0.0575	0.0574	0.000
σ^2	-0.2480	0.0608	NA	NA	-0.3696	0.0578	NA	NA	-0.5581	0.0575	NA	NA
ρ	-0.0246	0.0001	NA	NA	-0.0366	0.0001	NA	NA	-0.0599	0.0001	NA	NA

Table 4. Estimates of three parameter missing analyses under MCAR with missing rates in group1 (time1, time2) = (10%, 25%) and group2 (time1, time2) = (10%, 25%)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (group$_{(2-1)}$) = 0, σ^2 = 2, Correlation structure: Exchangeable

Correlation coefficient	ρ = 7				ρ = 2			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC								
β_0	0.0052	0.0736	0.0707	0.942	0.0003	0.0684	0.0670	0.946
β_1	0.0006	0.0433	0.0437	0.943	0.0023	0.0691	0.0715	0.966
β_2	-0.0092	0.1095	0.1063	0.929	-0.0016	0.0914	0.0895	0.936
σ^2	-0.0070	0.1095	NA	NA	0.0022	0.0914	NA	NA
ρ	-0.0003	0.0007	NA	NA	0.0011	0.0005	NA	NA

Table 4. Continue

LOCF								
β_0	0.0038	0.0626	0.0610	0.939	0.0020	0.0582	0.0575	0.940
β_1	0.0004	0.0271	0.0273	0.944	0.0015	0.0432	0.0447	0.966
β_2	-0.0042	0.0867	0.0851	0.940	-0.0024	0.0803	0.0774	0.944
σ^2	-0.0044	0.0867	NA	NA	-0.0007	0.0803	NA	NA
ρ	0.1125	0.0003	NA	NA	0.2996	0.0003	NA	NA
MI								
β_0	0.0046	0.0698	0.0600	0.910	0.0023	0.0649	0.0564	0.910
β_1	-0.0013	0.1043	0.0472	0.685	-0.0005	0.1176	0.0590	0.730
β_2	-0.0056	0.1057	0.0755	0.854	-0.0029	0.0968	0.0670	0.846
σ^2	-0.0166	0.1057	NA	NA	-0.0083	0.0968	NA	NA
ρ	-0.2601	0.0003	NA	NA	-0.0741	0.0003	NA	NA
EM								
β_0	0.0043	0.0616	0.0607	0.942	0.0022	0.0565	0.0566	0.942
β_1	0.0004	0.0462	0.0346	0.856	0.0009	0.0689	0.0566	0.895
β_2	-0.0050	0.0828	0.0824	0.945	-0.0028	0.0704	0.0694	0.940
σ^2	-0.0047	0.0828	NA	NA	0.0020	0.0704	NA	NA
ρ	0.0003	0.0003	NA	NA	0.0009	0.0003	NA	NA
LMM								
β_0	0.0040	0.0632	0.0617	0.938	0.0018	0.0594	0.0586	0.941
β_1	0.0004	0.0418	0.0426	0.948	0.0013	0.0643	0.0662	0.958
β_2	-0.0046	0.0869	0.0850	0.939	-0.0020	0.0784	0.0757	0.933
σ^2	-0.0007	0.0869	NA	NA	0.0044	0.0784	NA	NA
ρ	-0.1208	0.0003	NA	NA	-0.1354	0.0003	NA	NA
GEE								
β_0	0.0040	0.0632	0.0616	0.939	0.0018	0.0594	0.0586	0.940
β_1	0.0004	0.0418	0.0426	0.946	0.0013	0.0643	0.0662	0.957
β_2	-0.0046	0.0870	0.0849	0.939	-0.0020	0.0784	0.0756	0.932
σ^2	-0.0048	0.0870	NA	NA	0.0007	0.0784	NA	NA
ρ	-0.0001	0.0003	NA	NA	0.0017	0.0003	NA	NA
PROPOSED								
β_0	0.0040	0.0632	0.0617	0.939	0.0018	0.0594	0.0586	0.941
β_1	0.0004	0.0418	0.0426	0.947	0.0013	0.0643	0.0662	0.958
β_2	-0.0046	0.0869	0.0850	0.940	-0.0020	0.0784	0.0757	0.933
σ^2	-0.0007	0.0869	NA	NA	0.0045	0.0784	NA	NA
ρ	0.0001	0.0003	NA	NA	0.0019	0.0003	NA	NA

Table 5.1. Estimates of three parameter missing analyses under MAR with missing rate in (group1,group2)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = 0, β_2 (time$_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.7$, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0221	0.0607	0.0593	0.927	0.0023	0.0442	0.0447	0.955
β_1	-0.1519	0.0469	0.0424	0.049	-0.2753	0.0572	0.0539	0.001
β_2	-0.0402	0.0782	0.0757	0.909	-0.2789	0.0642	0.0576	0.004
σ^2	-0.1191	0.0782	NA	NA	-0.3863	0.0642	NA	NA
ρ	-0.1771	0.0003	NA	NA	-0.0643	0.0002	NA	NA
EM								
β_0	0.0011	0.0617	0.0607	0.938	0.0023	0.0442	0.0447	0.955
β_1	-0.0017	0.0362	0.0346	0.936	-0.2656	0.0532	0.0533	0.000
β_2	0.0018	0.0829	0.0824	0.942	-0.2735	0.0586	0.0575	0.003
σ^2	-0.0042	0.0829	NA	NA	-0.3864	0.0586	NA	NA
ρ	-0.0006	0.0003	NA	NA	-0.0435	0.0002	NA	NA
LMM								
β_0	0.0010	0.0619	0.0610	0.936	-0.0005	0.0442	0.0404	0.927
β_1	-0.0015	0.0351	0.0358	0.954	-0.2629	0.0524	0.0539	0.001
β_2	0.0020	0.0833	0.0829	0.945	-0.2754	0.0579	0.0572	0.001
σ^2	-0.0004	0.0833	NA	NA	-0.3693	0.0579	NA	NA
ρ	-0.1241	0.0003	NA	NA	-0.0546	0.0001	NA	NA
GEE								
β_0	0.0046	0.0617	0.0606	0.941	-0.0007	0.0443	0.0447	0.957
β_1	-0.0279	0.0363	0.0364	0.884	-0.2632	0.0524	0.0547	0.001
β_2	-0.0051	0.0822	0.0817	0.945	-0.2758	0.0580	0.0588	0.001
σ^2	-0.1081	0.0822	NA	NA	-0.3725	0.0580	NA	NA
ρ	-0.1225	0.0003	NA	NA	-0.0473	0.0002	NA	NA

Table 5.1. Continue

PROPOSED								
β_0	0.0025	0.0618	0.0589	0.933	0.0023	0.0442	0.0404	0.928
β_1	-0.0126	0.0356	0.0376	0.957	-0.2642	0.0524	0.0539	0.001
β_2	-0.0010	0.0827	0.0794	0.936	-0.2731	0.0578	0.0579	0.002
σ^2	-0.1121	0.0827	NA	NA	-0.3696	0.0578	NA	NA
ρ	-0.0514	0.0003	NA	NA	-0.0366	0.0001	NA	NA

Table 5.2. Estimates of three parameter missing analyses under MAR with missing rate in (group1,group2)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2\text{-}1)}$) = 0, β_2 (time$_{(3\text{-}1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0044	0.0555	0.0562	0.956	0.0113	0.0594	0.0559	0.933
β_1	-0.0426	0.0620	0.0585	0.877	-0.0807	0.0774	0.0593	0.675
β_2	-0.0125	0.0691	0.0674	0.936	-0.0309	0.0762	0.0662	0.886
σ^2	-0.0102	0.0691	NA	NA	-0.0246	0.0762	NA	NA
ρ	-0.0587	0.0002	NA	NA	-0.0913	0.0003	NA	NA
EM								
β_0	-0.0014	0.0552	0.0565	0.958	-0.0026	0.0567	0.0565	0.955
β_1	-0.0010	0.0595	0.0566	0.939	0.0034	0.0656	0.0564	0.906
β_2	-0.0009	0.0691	0.0692	0.949	-0.0031	0.0682	0.0692	0.948
σ^2	-0.0006	0.0691	NA	NA	-0.0079	0.0682	NA	NA
ρ	-0.0025	0.0002	NA	NA	0.0012	0.0003	NA	NA
LMM								
β_0	-0.0014	0.0555	0.0569	0.960	-0.0029	0.0579	0.0573	0.948
β_1	-0.0004	0.0583	0.0580	0.947	0.0037	0.0615	0.0600	0.938
β_2	-0.0009	0.0703	0.0704	0.950	-0.0025	0.0722	0.0719	0.949
σ^2	0.0031	0.0703	NA	NA	-0.0040	0.0722	NA	NA
ρ	-0.1381	0.0002	NA	NA	-0.1270	0.0003	NA	NA
GEE								
β_0	-0.0001	0.0555	0.0568	0.964	0.0013	0.0577	0.0571	0.950
β_1	-0.0098	0.0585	0.0583	0.945	-0.0216	0.0615	0.0605	0.928
β_2	-0.0034	0.0700	0.0700	0.948	-0.0109	0.0716	0.0712	0.945
σ^2	-0.0090	0.0700	NA	NA	-0.0196	0.0716	NA	NA
ρ	-0.0442	0.0002	NA	NA	-0.0571	0.0002	NA	NA
PROPOSED								
β_0	-0.0006	0.0555	0.0566	0.959	0.0001	0.0576	0.0568	0.949
β_1	-0.0060	0.0584	0.0588	0.947	-0.0144	0.0616	0.0612	0.939
β_2	-0.0024	0.0700	0.0695	0.947	-0.0085	0.0712	0.0705	0.943
σ^2	-0.0063	0.0700	NA	NA	-0.0178	0.0712	NA	NA
ρ	-0.0268	0.0002	NA	NA	-0.0404	0.0002	NA	NA

Table 6.1. Estimates of three parameter missing analyses under MNAR with missing rate in (group1, group2)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2\text{-}1)}$) = 0, β_2 (time$_{(3\text{-}1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.7$, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI								
β_0	0.0305	0.0578	0.0588	0.914	0.0787	0.0654	0.0578	0.694
β_1	-0.2137	0.0428	0.0394	0.001	-0.4368	0.0554	0.0426	0.000
β_2	-0.0606	0.0802	0.0741	0.851	-0.1575	0.0838	0.0686	0.403
σ^2	-0.2407	0.0802	NA	NA	-0.3711	0.0838	NA	NA
ρ	-0.1409	0.0002	NA	NA	-0.2564	0.0002	NA	NA
EM								
β_0	0.0171	0.0579	0.0594	0.941	0.0416	0.0604	0.0585	0.890
β_1	-0.1229	0.0369	0.0346	0.071	-0.2622	0.0373	0.0347	0.000
β_2	-0.0338	0.0804	0.0775	0.917	-0.0833	0.0737	0.0746	0.802
σ^2	-0.1983	0.0804	NA	NA	-0.3083	0.0737	NA	NA
ρ	-0.0319	0.0003	NA	NA	-0.0568	0.0003	NA	NA
LMM								
β_0	0.0159	0.0582	0.0581	0.931	0.0374	0.0618	0.0571	0.883
β_1	-0.1126	0.0359	0.0356	0.127	-0.2288	0.0359	0.0373	0.000
β_2	-0.0315	0.0819	0.0787	0.913	-0.0749	0.0767	0.0776	0.852
σ^2	-0.1746	0.0819	NA	NA	-0.2439	0.0767	NA	NA
ρ	-0.1486	0.0003	NA	NA	-0.1610	0.0003	NA	NA

Table 6.1. Continue

GEE								
β_0	0.0173	0.0581	0.0597	0.942	0.0428	0.0616	0.0592	0.880
β_1	-0.1227	0.0359	0.0356	0.069	-0.2607	0.0355	0.0370	0.000
β_2	-0.0343	0.0816	0.0783	0.911	-0.0858	0.0759	0.0766	0.803
σ^2	-0.2276	0.0816	NA	NA	-0.3164	0.0759	NA	NA
ρ	-0.0916	0.0003	NA	NA	-0.1420	0.0003	NA	NA
PROPOSED								
β_0	0.0165	0.0582	0.0570	0.927	0.0400	0.0616	0.0555	0.863
β_1	-0.1167	0.0359	0.0365	0.113	-0.2440	0.0356	0.0390	0.000
β_2	-0.0326	0.0818	0.0769	0.909	-0.0801	0.0760	0.0749	0.818
σ^2	-0.2278	0.0818	NA	NA	-0.3226	0.0760	NA	NA
ρ	-0.0520	0.0003	NA	NA	-0.0870	0.0003	NA	NA

Table 6.2. Estimates of three parameter missing analyses under MNAR with missing rate in (group1, group2)

Gold standard: β_0 (Baseline) = 1, β_1 ($time_{(2-1)}$) = 0, β_2 ($time_{(3-1)}$) = 0, $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
MI								
β_0	0.0252	0.0568	0.0552	0.908	0.0817	0.0580	0.0545	0.669
β_1	-0.2087	0.0592	0.0544	0.044	-0.4400	0.0687	0.0536	0.000
β_2	-0.0568	0.0658	0.0639	0.850	-0.1569	0.0738	0.0606	0.294
σ^2	-0.2396	0.0658	NA	NA	-0.3654	0.0738	NA	NA
ρ	-0.0415	0.0002	NA	NA	-0.0771	0.0002	NA	NA
EM								
β_0	0.0229	0.0559	0.0552	0.915	0.0684	0.0540	0.0545	0.753
β_1	-0.2015	0.0567	0.0540	0.046	-0.4151	0.0582	0.0526	0.000
β_2	-0.0524	0.0630	0.0643	0.876	-0.1302	0.0595	0.0616	0.433
σ^2	-0.2395	0.0630	NA	NA	-0.3591	0.0595	NA	NA
ρ	-0.0270	0.0002	NA	NA	-0.0444	0.0002	NA	NA
LMM								
β_0	0.0236	0.0564	0.0534	0.904	0.0736	0.0551	0.0522	0.699
β_1	-0.2009	0.0558	0.0554	0.049	-0.4184	0.0559	0.0564	0.000
β_2	-0.0537	0.0643	0.0656	0.876	-0.1407	0.0631	0.0649	0.405
σ^2	-0.2271	0.0643	NA	NA	-0.3224	0.0631	NA	NA
ρ	-0.1539	0.0002	NA	NA	-0.1614	0.0002	NA	NA
GEE								
β_0	0.0237	0.0564	0.0556	0.913	0.0738	0.0551	0.0554	0.725
0_1	-0.2011	0.0557	0.0550	0.049	-0.4193	0.0559	0.0550	0.000
B_2	-0.0538	0.0643	0.0655	0.876	-0.1411	0.0631	0.0645	0.399
σ^2	-0.2305	0.0643	NA	NA	-0.3264	0.0631	NA	NA
ρ	-0.0294	0.0002	NA	NA	-0.0517	0.0002	NA	NA
PROPOSED								
β_0	0.0236	0.0564	0.0533	0.904	0.0736	0.0550	0.0522	0.698
β_1	-0.2010	0.0557	0.0554	0.049	-0.4184	0.0559	0.0564	0.000
β_2	-0.0538	0.0643	0.0655	0.876	-0.1407	0.0630	0.0648	0.401
σ^2	-0.2275	0.0643	NA	NA	-0.3225	0.0630	NA	NA
ρ	-0.0267	0.0002	NA	NA	-0.0429	0.0002	NA	NA

Table 7.1. Estimates of four parameter missing analyses under MCAR with missing rates of (time1, time2, time3, time4)

Gold standard: β_0 (Baseline) = 1, β_1 ($time_{(2-1)}$) = β_2 ($time_{(3-1)}$) = β_3 ($time_{(4-1)}$) $\sigma^2 = 2$, $\rho = 0.7$, Correlation structure: Exchangeable

Missing rate	(10%, 10%, 10%, 10%)				(25%, 25%, 25%, 25%)				(5%, 10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	0.0012	0.0514	0.0524	0.949	-0.0001	0.0666	0.0686	0.954	0.0015	0.0761	0.0767	0.942
β_1	-0.0015	0.0411	0.0406	0.943	-0.0010	0.0528	0.0532	0.940	-0.0015	0.0627	0.0595	0.928
β_2	-0.0016	0.0382	0.0405	0.954	-0.0011	0.0527	0.0532	0.952	-0.0034	0.0589	0.0594	0.947
β_3	0.0006	0.0401	0.0406	0.951	0.0013	0.0532	0.0532	0.954	0.0005	0.0600	0.0595	0.948
σ^2	-0.0004	0.0796	NA	NA	-0.0094	0.1107	NA	NA	-0.0117	0.1180	NA	NA
ρ	-0.0003	0.0142	NA	NA	-0.0014	0.0186	NA	NA	-0.0012	0.0202	NA	NA
LOCF												
β_0	0.0003	0.0439	0.0447	0.948	-0.0011	0.0447	0.0447	0.946	0.0026	0.0461	0.0447	0.937
β_1	-0.0018	0.0330	0.0328	0.951	-0.0006	0.0311	0.0300	0.934	0.0003	0.0349	0.0328	0.941
β_2	-0.0015	0.0324	0.0344	0.960	-0.0012	0.0348	0.0335	0.933	-0.0009	0.0340	0.0342	0.946
β_3	-0.0002	0.0343	0.0346	0.947	-0.0006	0.0349	0.0344	0.945	0.0007	0.0343	0.0344	0.949
σ^2	0.0004	0.0688	NA	NA	-0.0043	0.0727	NA	NA	-0.0025	0.0716	NA	NA
ρ	0.0162	0.0116	NA	NA	0.0439	0.0110	NA	NA	0.0505	0.0107	NA	NA

Table 7.1. Continue

MI												
β_0	0.0003	0.0439	0.0447	0.948	-0.0011	0.0447	0.0447	0.946	0.0026	0.0461	0.0447	0.937
β_1	0.0001	0.0426	0.0384	0.924	-0.0014	0.0650	0.0435	0.850	0.0001	0.0411	0.0384	0.927
β_2	-0.0009	0.0419	0.0384	0.934	-0.0014	0.0651	0.0435	0.832	0.0005	0.0659	0.0435	0.835
β_3	0.0009	0.0486	0.0385	0.887	0.0002	0.0821	0.0435	0.760	0.0027	0.1502	0.0506	0.598
σ^2	-0.0008	0.0775	NA	NA	-0.0082	0.1063	NA	NA	-0.0131	0.1154	NA	NA
ρ	-0.1010	0.0152	NA	NA	-0.2386	0.0205	NA	NA	-0.2696	0.0221	NA	NA
EM												
β_0	0.0003	0.0439	0.0447	0.948	-0.0011	0.0447	0.0447	0.946	0.0026	0.0461	0.0447	0.937
β_1	-0.0014	0.0369	0.0346	0.934	-0.0016	0.0417	0.0346	0.902	-0.0004	0.0388	0.0346	0.917
β_2	-0.0012	0.0351	0.0346	0.945	-0.0025	0.0419	0.0346	0.887	-0.0020	0.0401	0.0346	0.910
β_3	0.0000	0.0370	0.0347	0.930	-0.0001	0.0415	0.0346	0.889	0.0002	0.0466	0.0346	0.856
σ^2	0.0001	0.0689	NA	NA	-0.0050	0.0755	NA	NA	-0.0021	0.0743	NA	NA
ρ	0.0000	0.0127	NA	NA	-0.0007	0.0134	NA	NA	-0.0001	0.0136	NA	NA
LMM												
β_0	0.0003	0.0439	0.0447	0.952	-0.0011	0.0447	0.0447	0.946	0.0026	0.0461	0.0447	0.938
β_1	-0.0017	0.0360	0.0359	0.954	-0.0011	0.0397	0.0384	0.936	0.0003	0.0376	0.0359	0.937
β_2	-0.0014	0.0337	0.0359	0.964	-0.0014	0.0393	0.0384	0.937	-0.0013	0.0375	0.0384	0.955
β_3	0.0000	0.0357	0.0359	0.943	-0.0003	0.0388	0.0384	0.943	0.0010	0.0437	0.0447	0.955
σ^2	0.0023	0.0682	NA	NA	-0.0029	0.0727	NA	NA	-0.0002	0.0703	NA	NA
ρ	-0.3367	0.0164	NA	NA	-0.3373	0.0182	NA	NA	-0.3360	0.0175	NA	NA
GEE												
β_0	0.0003	0.0439	0.0447	0.948	-0.0011	0.0447	0.0447	0.946	0.0026	0.0461	0.0447	0.937
β_1	-0.0017	0.0360	0.0359	0.954	-0.0011	0.0397	0.0383	0.934	0.0003	0.0376	0.0359	0.937
β_2	-0.0014	0.0337	0.0358	0.965	-0.0014	0.0393	0.0383	0.939	-0.0013	0.0375	0.0384	0.950
β_3	0.0000	0.0357	0.0359	0.946	-0.0003	0.0388	0.0383	0.943	0.0010	0.0437	0.0447	0.958
σ^2	0.0000	0.0686	NA	NA	-0.0052	0.0737	NA	NA	-0.0028	0.0712	NA	NA
ρ	0.0000	0.0137	NA	NA	-0.0010	0.0170	NA	NA	-0.0006	0.0169	NA	NA
PROPOSED												
β_0	0.0003	0.0439	0.0447	0.952	-0.0011	0.0447	0.0447	0.946	0.0026	0.0461	0.0447	0.940
β_1	-0.0017	0.0360	0.0359	0.953	-0.0011	0.0397	0.0384	0.937	0.0003	0.0376	0.0360	0.937
β_2	-0.0014	0.0337	0.0359	0.964	-0.0014	0.0393	0.0384	0.940	-0.0013	0.0375	0.0384	0.953
β_3	0.0000	0.0357	0.0359	0.945	-0.0003	0.0388	0.0384	0.943	0.0010	0.0437	0.0448	0.955
σ^2	0.0016	0.0751	NA	NA	-0.0026	0.0920	NA	NA	-0.0007	0.0945	NA	NA
ρ	-0.0003	0.0142	NA	NA	-0.0014	0.0186	NA	NA	-0.0012	0.0202	NA	NA

Table 7.2. Estimates of four parameter missing analyses in MCAR with missing rates of (time1, time2, time3, time4)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2\text{-}1)}$) = β_2 (time$_{(3\text{-}1)}$) = β_3 (time$_{(4\text{-}1)}$) σ^2 = 2, ρ = 0.7, Correlation structure: AR (1)

Missing rate	(10%, 10%, 10%, 10%)				(25%, 25%, 25%, 25%)				(5%, 10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	0.0006	0.0543	0.0552	0.941	-0.0021	0.0825	0.0793	0.935	0.0012	0.0792	0.0789	0.950
β_1	-0.0001	0.0434	0.0427	0.951	0.0013	0.0607	0.0616	0.943	-0.0007	0.0610	0.0611	0.948
β_2	-0.0034	0.0585	0.0557	0.931	0.0024	0.0782	0.0802	0.953	-0.0010	0.0796	0.0796	0.945
β_3	-0.0030	0.0648	0.0632	0.944	0.0035	0.0921	0.0911	0.942	-0.0013	0.0874	0.0904	0.956
σ^2	-0.0050	0.0777	NA	NA	-0.0085	0.1149	NA	NA	-0.0041	0.1114	NA	NA
ρ	-0.0007	0.0160	NA	NA	-0.0018	0.0232	NA	NA	-0.0007	0.0233	NA	NA
LOCF												
β_0	0.0004	0.0461	0.0459	0.947	0.0000	0.0498	0.0481	0.943	0.0006	0.0466	0.0453	0.954
β_1	-0.0002	0.0347	0.0344	0.950	0.0009	0.0340	0.0342	0.956	-0.0001	0.0341	0.0336	0.944
β_2	-0.0028	0.0470	0.0452	0.931	0.0018	0.0452	0.0452	0.954	-0.0003	0.0418	0.0430	0.953
β_3	-0.0032	0.0534	0.0516	0.940	0.0015	0.0520	0.0519	0.945	-0.0005	0.0471	0.0476	0.961
σ^2	-0.0019	0.0664	NA	NA	-0.0030	0.0707	NA	NA	0.0002	0.0694	NA	NA
ρ	0.0132	0.0132	NA	NA	0.0397	0.0133	NA	NA	0.0662	0.0113	NA	NA
MI												
β_0	0.0026	0.0548	0.0447	0.889	0.0015	0.0865	0.0446	0.738	0.0007	0.0498	0.0447	0.922
β_1	-0.0017	0.0416	0.0416	0.946	0.0002	0.0553	0.0490	0.921	0.0000	0.0393	0.0400	0.954
β_2	-0.0042	0.0544	0.0490	0.917	0.0014	0.0731	0.0536	0.859	-0.0007	0.0652	0.0509	0.885
β_3	-0.0047	0.0627	0.0537	0.901	0.0000	0.0896	0.0566	0.822	-0.0044	0.1489	0.0575	0.666
σ^2	-0.0038	0.0752	NA	NA	-0.0109	0.1240	NA	NA	-0.0121	0.1229	NA	NA
ρ	-0.0914	0.0167	NA	NA	-0.2404	0.0248	NA	NA	-0.2086	0.0233	NA	NA

Table 7.2. Continue

EM												
β_0	0.0000	0.0470	0.0447	0.936	0.0002	0.0535	0.0447	0.903	0.0008	0.0474	0.0447	0.946
β_1	0.0003	0.0398	0.0346	0.906	0.0010	0.0478	0.0347	0.844	-0.0003	0.0390	0.0346	0.909
β_2	-0.0030	0.0502	0.0451	0.913	0.0013	0.0570	0.0452	0.884	-0.0014	0.0506	0.0451	0.922
β_3	-0.0024	0.0569	0.0512	0.917	0.0014	0.0633	0.0513	0.895	-0.0001	0.0672	0.0512	0.863
σ^2	-0.0016	0.0664	NA	NA	-0.0022	0.0711	NA	NA	-0.0007	0.0707	NA	NA
ρ	-0.0002	0.0138	NA	NA	-0.0005	0.0156	NA	NA	0.0003	0.0158	NA	NA
LMM												
β_0	0.0003	0.0468	0.0458	0.949	0.0001	0.0510	0.0484	0.951	0.0006	0.0472	0.0452	0.950
β_1	0.0001	0.0388	0.0377	0.948	0.0012	0.0432	0.0438	0.956	-0.0002	0.0380	0.0374	0.939
β_2	-0.0030	0.0498	0.0465	0.926	0.0018	0.0534	0.0505	0.933	-0.0007	0.0488	0.0484	0.944
β_3	-0.0031	0.0560	0.0522	0.930	0.0012	0.0598	0.0558	0.932	-0.0007	0.0646	0.0596	0.934
σ^2	0.0002	0.0641	NA	NA	-0.0005	0.0683	NA	NA	0.0020	0.0665	NA	NA
ρ	-0.0290	0.0239	NA	NA	-0.0549	0.0298	NA	NA	-0.0397	0.0305	NA	NA
GEE												
β_0	0.0003	0.0468	0.0464	0.950	0.0001	0.0511	0.0496	0.953	0.0006	0.0473	0.0457	0.951
β_1	0.0001	0.0388	0.0377	0.948	0.0011	0.0433	0.0435	0.954	-0.0002	0.0381	0.0369	0.934
β_2	-0.0030	0.0498	0.0482	0.936	0.0018	0.0535	0.0536	0.944	-0.0007	0.0489	0.0498	0.947
β_3	-0.0031	0.0561	0.0546	0.940	0.0011	0.0600	0.0604	0.957	-0.0007	0.0647	0.0636	0.948
σ^2	-0.0021	0.0655	NA	NA	-0.0023	0.0691	NA	NA	0.0003	0.0678	NA	NA
ρ	-0.0100	0.0151	NA	NA	-0.0256	0.0189	NA	NA	-0.0146	0.0176	NA	NA
PROPOSED												
β_0	0.0004	0.0462	0.0460	0.947	0.0000	0.0505	0.0488	0.949	0.0006	0.0467	0.0454	0.955
β_1	0.0000	0.0382	0.0374	0.954	0.0012	0.0425	0.0428	0.954	-0.0001	0.0374	0.0366	0.939
β_2	-0.0032	0.0491	0.0473	0.930	0.0018	0.0522	0.0520	0.947	-0.0006	0.0474	0.0489	0.955
β_3	-0.0034	0.0553	0.0535	0.942	0.0014	0.0580	0.0583	0.951	-0.0005	0.0621	0.0618	0.950
σ^2	0.0003	0.0684	NA	NA	0.0008	0.0857	NA	NA	0.0038	0.0841	NA	NA
ρ	-0.0004	0.0127	NA	NA	-0.0013	0.0194	NA	NA	-0.0008	0.0188	NA	NA

Table 7.3. Estimates of four parameter missing analyses under MCAR with missing rates of (time1, time2, time3, time4)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = β_2 (time$_{(3-1)}$) = β_3 (time$_{(4-1)}$) $\sigma^2 = 2$, $\rho = 0.2$, Correlation structure: Exchangeable

Missing rate	(10%, 10%, 10%, 10%)				(25%, 25%, 25%, 25%)				(5%, 10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	0.0031	0.0526	0.0523	0.949	0.0007	0.0716	0.0688	0.941	0.0041	0.0736	0.0769	0.959
β_1	-0.0047	0.0658	0.0662	0.954	-0.0001	0.0869	0.0868	0.947	-0.0042	0.0956	0.0970	0.951
β_2	-0.0038	0.0674	0.0662	0.942	-0.0032	0.0867	0.0871	0.955	-0.0028	0.0959	0.0974	0.956
β_3	-0.0037	0.0635	0.0662	0.955	-0.0018	0.0888	0.0870	0.940	-0.0059	0.0940	0.0973	0.958
σ^2	-0.0020	0.0546	NA	NA	-0.0027	0.0717	NA	NA	-0.0028	0.0804	NA	NA
ρ	0.0008	0.0185	NA	NA	0.0005	0.0254	NA	NA	-0.0001	0.0289	NA	NA
LOCF												
β_0	0.0028	0.0448	0.0447	0.950	-0.0002	0.0444	0.0447	0.957	0.0018	0.0463	0.0447	0.945
β_1	-0.0033	0.0526	0.0537	0.953	0.0007	0.0488	0.0489	0.949	-0.0037	0.0538	0.0536	0.954
β_2	-0.0027	0.0567	0.0563	0.943	-0.0025	0.0547	0.0547	0.950	-0.0019	0.0565	0.0558	0.946
β_3	-0.0014	0.0565	0.0565	0.955	-0.0015	0.0563	0.0561	0.950	-0.0024	0.0553	0.0562	0.946
σ^2	-0.0012	0.0499	NA	NA	-0.0017	0.0572	NA	NA	-0.0033	0.0566	NA	NA
ρ	0.0436	0.0163	NA	NA	0.1187	0.0176	NA	NA	0.1342	0.0183	NA	NA
MI												
β_0	0.0028	0.0448	0.0447	0.950	-0.0002	0.0444	0.0447	0.957	0.0018	0.0463	0.0447	0.945
β_1	-0.0045	0.0581	0.0573	0.944	-0.0023	0.0743	0.0581	0.894	-0.0037	0.0597	0.0572	0.946
β_2	-0.0040	0.0616	0.0572	0.925	-0.0039	0.0772	0.0582	0.884	-0.0011	0.0777	0.0582	0.876
β_3	-0.0041	0.0644	0.0571	0.936	-0.0007	0.0945	0.0581	0.798	-0.0040	0.1624	0.0595	0.606
σ^2	-0.0023	0.0592	NA	NA	-0.0100	0.0930	NA	NA	-0.0175	0.1134	NA	NA
ρ	-0.0281	0.0150	NA	NA	-0.0671	0.0145	NA	NA	-0.0766	0.0144	NA	NA
EM												
β_o	0.0028	0.0448	0.0447	0.950	-0.0002	0.0444	0.0447	0.957	0.0018	0.0463	0.0447	0.945
β_1	-0.0049	0.0591	0.0566	0.932	0.0016	0.0635	0.0564	0.918	-0.0038	0.0601	0.0565	0.929
β_2	-0.0031	0.0609	0.0566	0.933	-0.0032	0.0643	0.0565	0.920	-0.0012	0.0660	0.0565	0.906
β_3	-0.0021	0.0587	0.0564	0.939	-0.0013	0.0646	0.0565	0.910	-0.0048	0.0760	0.0565	0.860
σ^2	-0.0011	0.0502	NA	NA	-0.0022	0.0581	NA	NA	-0.0037	0.0576	NA	NA
ρ	0.0009	0.0176	NA	NA	0.0000	0.0211	NA	NA	-0.0011	0.0227	NA	NA
LMM												
β_0	0.0028	0.0448	0.0447	0.951	-0.0002	0.0444	0.0447	0.955	0.0018	0.0463	0.0447	0.949
β_1	-0.0041	0.0572	0.0583	0.948	0.0012	0.0610	0.0618	0.954	-0.0040	0.0589	0.0584	0.952
β_2	-0.0025	0.0593	0.0583	0.946	-0.0034	0.0603	0.0618	0.966	-0.0010	0.0620	0.0618	0.943
β_3	-0.0019	0.0574	0.0583	0.954	-0.0001	0.0617	0.0618	0.957	-0.0041	0.0696	0.0710	0.953
σ^2	0.0006	0.0483	NA	NA	0.0005	0.0531	NA	NA	-0.0011	0.0525	NA	NA
ρ	-0.5423	0.0123	NA	NA	-0.5318	0.0190	NA	NA	-0.5228	0.0231	NA	NA

Table 7.3. Continue

GEE												
β_0	0.0028	0.0448	0.0447	0.950	-0.0002	0.0444	0.0447	0.957	0.0018	0.0463	0.0447	0.945
β_1	-0.0041	0.0572	0.0584	0.949	0.0012	0.0610	0.0616	0.953	-0.0040	0.0589	0.0583	0.955
β_2	-0.0025	0.0593	0.0583	0.945	-0.0034	0.0603	0.0618	0.965	-0.0010	0.0620	0.0618	0.942
β_3	-0.0019	0.0574	0.0582	0.956	-0.0001	0.0617	0.0618	0.958	-0.0041	0.0696	0.0709	0.952
σ^2	-0.0015	0.0483	NA	NA	-0.0019	0.0530	NA	NA	-0.0035	0.0525	NA	NA
ρ	0.0008	0.0169	NA	NA	0.0003	0.0196	NA	NA	-0.0005	0.0201	NA	NA
PROPOSED												
β_0	0.0028	0.0448	0.0447	0.951	-0.0002	0.0444	0.0447	0.955	0.0018	0.0463	0.0447	0.949
β_1	-0.0041	0.0572	0.0583	0.948	0.0012	0.0610	0.0618	0.955	-0.0040	0.0589	0.0584	0.952
β_2	-0.0025	0.0593	0.0583	0.945	-0.0033	0.0603	0.0618	0.965	-0.0010	0.0620	0.0618	0.943
β_3	-0.0019	0.0574	0.0583	0.953	-0.0001	0.0617	0.0618	0.974	-0.0041	0.0696	0.0710	0.954
σ^2	0.0009	0.0490	NA	NA	0.0019	0.0543	NA	NA	0.0011	0.0551	NA	NA
ρ	0.0008	0.0185	NA	NA	0.0005	0.0254	NA	NA	-0.0001	0.0289	NA	NA

Table 7.4. Estimates of four parameter missing analyses under MCAR with missing rates of (time1, time2, time3, time4)

Gold standard: β_0 (Baseline) = 1, β_1 (time$_{(2-1)}$) = β_2 (time$_{(3-1)}$) = β_3 (time$_{(4-1)}$) σ^2 = 2, ρ = 0.2, Correlation structure: AR (1)

Missing rate	(10%, 10%, 10%, 10%)				(25%, 25%, 25%, 25%)				(5%, 10%, 25%, 50%)			
Parameter	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP	Bias	Std. Dev	SE	CP
CC												
β_0	-0.0023	0.0537	0.0552	0.959	0.0017	0.0766	0.0795	0.954	0.0014	0.0791	0.0788	0.955
β_1	0.0019	0.0670	0.0698	0.957	-0.0017	0.0987	0.1004	0.950	0.0017	0.0989	0.0997	0.955
β_2	-0.0017	0.0760	0.0764	0.952	-0.0049	0.1076	0.1101	0.958	-0.0017	0.1105	0.1091	0.946
β_3	0.0053	0.0781	0.0777	0.946	-0.0038	0.1079	0.1119	0.949	-0.0002	0.1144	0.1111	0.949
σ^2	-0.0071	0.0582	NA	NA	-0.0070	0.0855	NA	NA	-0.0099	0.0792	NA	NA
ρ	-0.0011	0.0228	NA	NA	-0.0008	0.0337	NA	NA	-0.0003	0.0326	NA	NA
LOCF												
β_0	-0.0018	0.0471	0.0469	0.947	0.0013	0.0502	0.0508	0.948	-0.0011	0.0470	0.0458	0.941
β_1	0.0019	0.0537	0.0556	0.956	-0.0027	0.0548	0.0548	0.945	0.0010	0.0567	0.0546	0.939
β_2	-0.0007	0.0637	0.0627	0.944	-0.0029	0.0643	0.0638	0.942	0.0014	0.0614	0.0608	0.952
β_3	0.0034	0.0659	0.0644	0.944	-0.0021	0.0678	0.0666	0.942	0.0013	0.0634	0.0623	0.943
σ^2	-0.0062	0.0502	NA	NA	-0.0018	0.0565	NA	NA	-0.0062	0.0549	NA	NA
ρ	0.0660	0.0187	NA	NA	0.1712	0.0207	NA	NA	0.2082	0.0205	NA	NA
MI												
β_0	-0.0027	0.0556	0.0447	0.890	0.0019	0.0952	0.0446	0.715	-0.0011	0.0494	0.0447	0.919
β_1	0.0021	0.0571	0.0578	0.950	-0.0012	0.0712	0.0593	0.898	0.0017	0.0585	0.0575	0.946
β_2	-0.0001	0.0676	0.0621	0.929	-0.0006	0.0917	0.0623	0.840	0.0014	0.0772	0.0622	0.889
β_3	0.0038	0.0728	0.0630	0.902	-0.0009	0.1041	0.0628	0.805	-0.0017	0.1717	0.0626	0.578
σ^2	-0.0082	0.0657	NA	NA	-0.0131	0.1197	NA	NA	-0.0246	0.1113	NA	NA
ρ	-0.0367	0.0172	NA	NA	-0.0835	0.0160	NA	NA	-0.0696	0.0174	NA	NA
EM												
β_0	-0.0014	0.0487	0.0447	0.930	0.0016	0.0556	0.0447	0.887	-0.0009	0.0479	0.0447	0.926
β_1	0.0015	0.0618	0.0565	0.928	-0.0034	0.0743	0.0566	0.869	0.0013	0.0637	0.0565	0.921
β_2	-0.0017	0.0690	0.0619	0.925	-0.0033	0.0788	0.0620	0.873	0.0026	0.0718	0.0620	0.912
β_3	0.0024	0.0698	0.0630	0.915	-0.0023	0.0791	0.0630	0.880	0.0001	0.0855	0.0630	0.843
σ^2	-0.0055	0.0517	NA	NA	-0.0034	0.0563	NA	NA	-0.0050	0.0560	NA	NA
ρ	-0.0008	0.0210	NA	NA	-0.0011	0.0260	NA	NA	-0.0011	0.0264	NA	NA
LMM												
β_0	-0.0019	0.0473	0.0470	0.949	0.0017	0.0509	0.0514	0.949	-0.0010	0.0471	0.0458	0.941
β_1	0.0023	0.0583	0.0604	0.953	-0.0039	0.0685	0.0675	0.938	0.0013	0.0615	0.0596	0.945
β_2	-0.0008	0.0665	0.0646	0.940	-0.0032	0.0739	0.0705	0.936	0.0016	0.0682	0.0670	0.943
β_3	0.0037	0.0675	0.0657	0.945	-0.0026	0.0731	0.0716	0.943	0.0002	0.0796	0.0767	0.941
σ^2	-0.0037	0.0491	NA	NA	-0.0007	0.0522	NA	NA	-0.0032	0.0497	NA	NA
ρ	-0.0207	0.0234	NA	NA	-0.0403	0.0283	NA	NA	-0.0275	0.0269	NA	NA
GEE												
β_0	-0.0019	0.0473	0.0471	0.950	0.0017	0.0509	0.0516	0.950	-0.0010	0.0471	0.0459	0.941
β_1	0.0023	0.0582	0.0602	0.951	-0.0039	0.0685	0.0670	0.939	0.0012	0.0614	0.0593	0.942
β_2	-0.0008	0.0665	0.0652	0.944	-0.0032	0.0740	0.0717	0.941	0.0015	0.0682	0.0677	0.949
β_3	0.0037	0.0675	0.0664	0.947	-0.0026	0.0732	0.0727	0.948	0.0003	0.0797	0.0777	0.943
σ^2	-0.0059	0.0490	NA	NA	-0.0033	0.0522	NA	NA	-0.0058	0.0496	NA	NA
ρ	-0.0110	0.0194	NA	NA	-0.0266	0.0229	NA	NA	-0.0161	0.0225	NA	NA
PROPOSED												
β_0	-0.0018	0.0471	0.0470	0.950	0.0015	0.0510	0.0514	0.950	-0.0010	0.0471	0.0458	0.940
β_1	0.0021	0.0582	0.0601	0.952	-0.0037	0.0684	0.0669	0.942	0.0012	0.0614	0.0592	0.944
β_2	-0.0009	0.0663	0.0652	0.945	-0.0029	0.0736	0.0715	0.941	0.0015	0.0681	0.0676	0.945
β_3	0.0036	0.0673	0.0662	0.949	-0.0023	0.0729	0.0725	0.951	0.0004	0.0793	0.0775	0.945
σ^2	-0.0035	0.0494	NA	NA	0.0008	0.0536	NA	NA	-0.0014	0.0512	NA	NA
ρ	-0.0011	0.0216	NA	NA	-0.0005	0.0316	NA	NA	-0.0004	0.0306	NA	NA

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Table 8.1. Results of four parameter missing analyses under MAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure : exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	-0.0004	0.0445	0.0447	0.953	0.0007	0.0455	0.0447	0.950	0.0002	0.0464	0.0448	0.944
β_1	-0.1047	0.0403	0.0406	0.252	-0.1890	0.0447	0.0436	0.006	-0.4126	0.0762	0.0491	0.000
β_2	-0.1063	0.0380	0.0388	0.217	-0.1934	0.0438	0.0413	0.001	-0.4172	0.0724	0.0461	0.001
β_3	-0.1072	0.0400	0.0388	0.221	-0.1954	0.0481	0.0414	0.010	-0.4246	0.0837	0.0462	0.001
σ^2	-0.1397	0.0626	NA	NA	-0.2107	0.0634	NA	NA	-0.3420	0.0860	NA	NA
ρ	-0.1254	0.0176	NA	NA	-0.1995	0.0266	NA	NA	-0.3479	0.0424	NA	NA
EM												
β_0	-0.0004	0.0445	0.0447	0.953	0.0007	0.0455	0.0447	0.950	0.0002	0.0464	0.0448	0.944
β_1	-0.0033	0.0354	0.0347	0.946	-0.0086	0.0371	0.0348	0.923	-0.0256	0.0480	0.0351	0.778
β_2	-0.0091	0.0356	0.0348	0.936	-0.0224	0.0376	0.0350	0.894	-0.0794	0.0454	0.0356	0.413
β_3	-0.0076	0.0373	0.0347	0.921	-0.0194	0.0371	0.0348	0.899	-0.0683	0.0448	0.0352	0.497
σ^2	-0.0236	0.0723	NA	NA	-0.0453	0.0714	NA	NA	-0.1100	0.0759	NA	NA
ρ	-0.0050	0.0130	NA	NA	-0.0100	0.0133	NA	NA	-0.0259	0.0165	NA	NA
LMM												
β_0	-0.0004	0.0445	0.0445	0.952	0.0007	0.0455	0.0443	0.948	0.0002	0.0464	0.0438	0.938
β_1	-0.0035	0.0346	0.0354	0.952	-0.0096	0.0354	0.0362	0.939	-0.0298	0.0396	0.0391	0.868
β_2	-0.0083	0.0351	0.0354	0.945	-0.0194	0.0363	0.0362	0.925	-0.0596	0.0399	0.0391	0.676
β_3	-0.0071	0.0362	0.0354	0.942	-0.0175	0.0354	0.0362	0.924	-0.0557	0.0393	0.0391	0.712
σ^2	-0.0203	0.0713	NA	NA	-0.0388	0.0700	NA	NA	-0.0834	0.0742	NA	NA
ρ	-0.3431	0.0198	NA	NA	-0.3500	0.0219	NA	NA	-0.3731	0.0329	NA	NA
GEE												
β_0	-0.0004	0.0445	0.0447	0.953	0.0007	0.0455	0.0447	0.950	0.0002	0.0464	0.0448	0.944
β_1	-0.0108	0.0349	0.0356	0.936	-0.0280	0.0359	0.0366	0.888	-0.0708	0.0404	0.0398	0.575
β_2	-0.0152	0.0351	0.0355	0.932	-0.0368	0.0367	0.0365	0.831	-0.0978	0.0402	0.0397	0.300
β_3	-0.0141	0.0362	0.0354	0.929	-0.0352	0.0356	0.0364	0.844	-0.0946	0.0395	0.0393	0.326
σ^2	-0.1324	0.0625	NA	NA	-0.1876	0.0609	NA	NA	-0.2404	0.0649	NA	NA
ρ	-0.0801	0.0132	NA	NA	-0.1087	0.0139	NA	NA	-0.1157	0.0187	NA	NA
PROPOSED												
β_0	-0.0004	0.0445	0.0421	0.936	0.0007	0.0455	0.0411	0.925	0.0002	0.0464	0.0397	0.912
β_1	-0.0092	0.0348	0.0365	0.949	-0.0259	0.0358	0.0380	0.907	-0.0947	0.0413	0.0426	0.399
β_2	-0.0137	0.0351	0.0365	0.949	-0.0348	0.0367	0.0380	0.863	-0.1200	0.0410	0.0426	0.180
β_3	-0.0125	0.0362	0.0365	0.941	-0.0332	0.0357	0.0380	0.875	-0.1171	0.0407	0.0426	0.194
σ^2	-0.2286	0.0589	NA	NA	-0.3124	0.0556	NA	NA	-0.4277	0.0540	NA	NA
ρ	-0.0638	0.0147	NA	NA	-0.0977	0.0160	NA	NA	-0.1683	0.0216	NA	NA

Table 8.2. Results of four parameter missing analyses under MAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure : AR(1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	**SE**	**CP**
MI												
β_0	-0.0015	0.0448	0.0447	0.951	-0.0003	0.0444	0.0447	0.956	-0.0002	0.0450	0.0447	0.946
β_1	-0.1050	0.0408	0.0406	0.258	-0.1894	0.0471	0.0437	0.015	-0.4130	0.0795	0.0490	0.000
β_2	-0.1064	0.0487	0.0480	0.413	-0.1942	0.0531	0.0497	0.031	-0.4173	0.0856	0.0525	0.003
β_3	-0.1063	0.0536	0.0525	0.472	-0.1948	0.0590	0.0534	0.053	-0.4237	0.0926	0.0547	0.002
σ^2	-0.1423	0.0588	NA	NA	-0.2105	0.0571	NA	NA	-0.3460	0.0793	NA	NA
ρ	-0.0982	0.0189	NA	NA	-0.1668	0.0265	NA	NA	-0.3275	0.0505	NA	NA
EM												
β_0	-0.0015	0.0448	0.0447	0.951	-0.0003	0.0444	0.0447	0.956	-0.0002	0.0450	0.0447	0.946
β_1	-0.0061	0.0362	0.0348	0.929	-0.0150	0.0383	0.0349	0.906	-0.0463	0.0470	0.0354	0.668
β_2	-0.0189	0.0458	0.0455	0.928	-0.0471	0.0489	0.0460	0.807	-0.1557	0.0579	0.0473	0.137
β_3	-0.0184	0.0514	0.0515	0.942	-0.0450	0.0543	0.0519	0.846	-0.1640	0.0619	0.0530	0.164
σ^2	-0.0428	0.0665	NA	NA	-0.0763	0.0646	NA	NA	-0.1772	0.0656	NA	NA
ρ	-0.0123	0.0146	NA	NA	-0.0243	0.0155	NA	NA	-0.0648	0.0199	NA	NA

Table 8.2. Continue

LMM												
β_0	0.0198	0.0459	0.0438	0.912	0.0330	0.0455	0.0433	0.862	0.0465	0.0459	0.0422	0.776
β_1	-0.0306	0.0365	0.0360	0.864	-0.0576	0.0381	0.0372	0.655	-0.1262	0.0405	0.0410	0.126
β_2	-0.0793	0.0476	0.0457	0.586	-0.1434	0.0498	0.0465	0.139	-0.2941	0.0558	0.0493	0.000
β_3	-0.0927	0.0540	0.0513	0.539	-0.1651	0.0562	0.0519	0.132	-0.3514	0.0614	0.0543	0.000
σ^2	-0.0713	0.0628	NA	NA	-0.1091	0.0591	NA	NA	-0.1865	0.0614	NA	NA
ρ	-0.0420	0.0228	NA	NA	-0.0564	0.0212	NA	NA	-0.0934	0.0200	NA	NA
GEE												
β_0	0.0162	0.0457	0.0457	0.933	0.0250	0.0451	0.0459	0.921	0.0323	0.0457	0.0455	0.887
β_1	-0.0324	0.0366	0.0364	0.859	-0.0631	0.0383	0.0376	0.612	-0.1457	0.0413	0.0412	0.054
β_2	-0.0806	0.0476	0.0482	0.611	-0.1468	0.0495	0.0498	0.149	-0.3075	0.0555	0.0531	0.000
β_3	-0.0955	0.0537	0.0545	0.567	-0.1707	0.0555	0.0561	0.137	-0.3656	0.0601	0.0589	0.000
σ^2	-0.1390	0.0578	NA	NA	-0.1996	0.0537	NA	NA	-0.2887	0.0552	NA	NA
ρ	-0.0711	0.0146	NA	NA	-0.1057	0.0161	NA	NA	-0.1603	0.0228	NA	NA
PROPOSED												
β_0	-0.0015	0.0448	0.0428	0.940	-0.0003	0.0444	0.0421	0.939	-0.0002	0.0450	0.0410	0.927
β_1	-0.0099	0.0357	0.0363	0.937	-0.0262	0.0372	0.0377	0.894	-0.0876	0.0405	0.0418	0.438
β_2	-0.0217	0.0453	0.0465	0.931	-0.0546	0.0472	0.0476	0.799	-0.1696	0.0538	0.0513	0.092
β_3	-0.0246	0.0512	0.0521	0.941	-0.0602	0.0528	0.0530	0.794	-0.1987	0.0584	0.0563	0.071
σ^2	-0.1673	0.0569	NA	NA	-0.2291	0.0537	NA	NA	-0.3223	0.0550	NA	NA
ρ	-0.0486	0.0126	NA	NA	-0.0730	0.0145	NA	NA	-0.1248	0.0199	NA	NA

Table 8.3. Results of four parameter missing analyses under MAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure : Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	0.0022	0.0457	0.0447	0.948	-0.0012	0.0458	0.0447	0.935	-0.0013	0.0449	0.0448	0.949
β_1	-0.0331	0.0591	0.0580	0.916	-0.0544	0.0593	0.0588	0.845	-0.1098	0.0727	0.0601	0.549
β_2	-0.0312	0.0581	0.0570	0.901	-0.0543	0.0618	0.0574	0.819	-0.1190	0.0689	0.0585	0.480
β_3	-0.0339	0.0594	0.0571	0.903	-0.0566	0.0642	0.0574	0.803	-0.1262	0.0958	0.0584	0.446
σ^2	-0.0152	0.0501	NA	NA	-0.0200	0.0566	NA	NA	-0.0289	0.0820	NA	NA
ρ	-0.0362	0.0180	NA	NA	-0.0567	0.0201	NA	NA	-0.0961	0.0238	NA	NA
EM												
β_0	0.0022	0.0457	0.0447	0.948	-0.0012	0.0458	0.0447	0.935	-0.0013	0.0449	0.0448	0.949
β_1	-0.0033	0.0592	0.0565	0.948	-0.0021	0.0594	0.0566	0.929	-0.0028	0.0722	0.0567	0.889
β_2	-0.0029	0.0593	0.0566	0.942	-0.0069	0.0632	0.0567	0.922	-0.0356	0.0684	0.0572	0.857
β_3	-0.0054	0.0589	0.0566	0.947	-0.0070	0.0608	0.0566	0.928	-0.0287	0.0700	0.0567	0.857
σ^2	-0.0058	0.0501	NA	NA	-0.0056	0.0495	NA	NA	-0.0118	0.0567	NA	NA
ρ	-0.0018	0.0188	NA	NA	-0.0044	0.0206	NA	NA	-0.0133	0.0231	NA	NA
LMM												
β_0	0.0022	0.0457	0.0447	0.946	-0.0012	0.0458	0.0447	0.934	-0.0013	0.0449	0.0446	0.950
β_1	-0.0038	0.0584	0.0574	0.947	-0.0031	0.0574	0.0584	0.950	-0.0105	0.0618	0.0621	0.942
β_2	-0.0033	0.0585	0.0574	0.953	-0.0062	0.0610	0.0584	0.940	-0.0336	0.0626	0.0621	0.914
β_3	-0.0057	0.0573	0.0574	0.954	-0.0072	0.0584	0.0584	0.942	-0.0326	0.0629	0.0621	0.917
σ^2	-0.0037	0.0488	NA	NA	-0.0035	0.0483	NA	NA	-0.0107	0.0522	NA	NA
ρ	-0.5433	0.0142	NA	NA	-0.5412	0.0151	NA	NA	-0.5218	0.0308	NA	NA
GEE												
β_0	0.0022	0.0457	0.0447	0.948	-0.0012	0.0458	0.0447	0.935	-0.0013	0.0449	0.0448	0.949
β_1	-0.0062	0.0585	0.0575	0.947	-0.0085	0.0576	0.0586	0.953	-0.0177	0.0618	0.0622	0.937
β_2	-0.0055	0.0585	0.0575	0.952	-0.0109	0.0609	0.0585	0.933	-0.0391	0.0625	0.0623	0.904
β_3	-0.0079	0.0573	0.0575	0.952	-0.0121	0.0584	0.0584	0.938	-0.0384	0.0627	0.0619	0.901
σ^2	-0.0148	0.0479	NA	NA	-0.0166	0.0473	NA	NA	-0.0197	0.0517	NA	NA
ρ	-0.0225	0.0158	NA	NA	-0.0294	0.0165	NA	NA	-0.0296	0.0188	NA	NA
PROPOSED												
β_0	0.0022	0.0457	0.0444	0.945	-0.0012	0.0458	0.0443	0.934	-0.0013	0.0449	0.0442	0.948
β_1	-0.0075	0.0585	0.0582	0.952	-0.0130	0.0577	0.0595	0.953	-0.0429	0.0623	0.0638	0.905
β_2	-0.0067	0.0586	0.0582	0.953	-0.0149	0.0608	0.0595	0.938	-0.0587	0.0624	0.0638	0.857
β_3	-0.0092	0.0574	0.0582	0.954	-0.0162	0.0584	0.0595	0.936	-0.0587	0.0632	0.0638	0.856
σ^2	-0.0279	0.0469	NA	NA	-0.0345	0.0461	NA	NA	-0.0436	0.0500	NA	NA
ρ	-0.0333	0.0173	NA	NA	-0.0489	0.0186	NA	NA	-0.0780	0.0225	NA	NA

Table 8.4. Results of four parameter missing analyses under MAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: AR(1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	-0.0001	0.0435	0.0448	0.961	-0.0001	0.0453	0.0447	0.934	0.0017	0.0446	0.0447	0.950
β_1	-0.0292	0.0559	0.0580	0.933	-0.0516	0.0625	0.0587	0.838	-0.1093	0.0720	0.0599	0.551
β_2	-0.0309	0.0630	0.0622	0.926	-0.0537	0.0667	0.0622	0.841	-0.1188	0.0805	0.0623	0.522
β_3	-0.0281	0.0648	0.0629	0.922	-0.0532	0.0721	0.0629	0.826	-0.1239	0.0989	0.0628	0.473
σ^2	-0.0101	0.0486	NA	NA	-0.0193	0.0562	NA	NA	-0.0319	0.0800	NA	NA
ρ	-0.0462	0.0226	NA	NA	-0.0739	0.0296	NA	NA	-0.1227	0.0347	NA	NA
EM												
β_0	-0.0001	0.0435	0.0448	0.961	-0.0001	0.0453	0.0447	0.934	0.0017	0.0446	0.0447	0.950
β_1	0.0001	0.0561	0.0566	0.951	-0.0002	0.0620	0.0566	0.940	-0.0047	0.0715	0.0566	0.890
β_2	-0.0037	0.0639	0.0621	0.940	-0.0092	0.0673	0.0622	0.928	-0.0469	0.0781	0.0628	0.816
β_3	-0.0015	0.0641	0.0630	0.948	-0.0068	0.0681	0.0630	0.925	-0.0433	0.0781	0.0631	0.834
σ^2	-0.0001	0.0479	NA	NA	-0.0050	0.0504	NA	NA	-0.0135	0.0550	NA	NA
ρ	-0.0013	0.0223	NA	NA	-0.0052	0.0246	NA	NA	-0.0187	0.0291	NA	NA
LMM												
β_0	0.0033	0.0437	0.0446	0.959	0.0045	0.0453	0.0445	0.934	0.0063	0.0447	0.0444	0.946
β_1	-0.0075	0.0557	0.0582	0.957	-0.0160	0.0603	0.0596	0.946	-0.0539	0.0628	0.0640	0.877
β_2	-0.0324	0.0633	0.0625	0.926	-0.0549	0.0658	0.0633	0.837	-0.1180	0.0715	0.0664	0.565
β_3	-0.0308	0.0639	0.0634	0.923	-0.0537	0.0658	0.0641	0.868	-0.1197	0.0695	0.0671	0.585
σ^2	-0.0051	0.0462	NA	NA	-0.0124	0.0483	NA	NA	-0.0252	0.0496	NA	NA
ρ	-0.0347	0.0232	NA	NA	-0.0515	0.0250	NA	NA	-0.0839	0.0273	NA	NA
GEE												
β_0	0.0030	0.0437	0.0449	0.958	0.0039	0.0453	0.0448	0.934	0.0053	0.0446	0.0447	0.949
β_1	-0.0087	0.0557	0.0579	0.959	-0.0188	0.0604	0.0591	0.942	-0.0610	0.0630	0.0631	0.849
β_2	-0.0334	0.0634	0.0637	0.928	-0.0565	0.0657	0.0648	0.850	-0.1205	0.0711	0.0680	0.566
β_3	-0.0321	0.0640	0.0642	0.926	-0.0558	0.0657	0.0650	0.866	-0.1225	0.0691	0.0680	0.574
σ^2	-0.0101	0.0459	NA	NA	-0.0180	0.0480	NA	NA	-0.0303	0.0494	NA	NA
ρ	-0.0361	0.0185	NA	NA	-0.0559	0.0191	NA	NA	-0.0914	0.0211	NA	NA
PROPOSED												
β_0	-0.0001	0.0435	0.0446	0.963	-0.0001	0.0453	0.0445	0.934	0.0017	0.0446	0.0444	0.949
β_1	-0.0039	0.0555	0.0582	0.958	-0.0100	0.0599	0.0594	0.944	-0.0420	0.0643	0.0636	0.905
β_2	-0.0073	0.0627	0.0629	0.953	-0.0171	0.0647	0.0638	0.948	-0.0654	0.0714	0.0671	0.825
β_3	-0.0049	0.0634	0.0637	0.950	-0.0148	0.0658	0.0645	0.933	-0.0640	0.0707	0.0677	0.832
σ^2	-0.0111	0.0458	NA	NA	-0.0189	0.0480	NA	NA	-0.0300	0.0494	NA	NA
ρ	-0.0259	0.0195	NA	NA	-0.0387	0.0213	NA	NA	-0.0652	0.0283	NA	NA

Table 9.1. Results of four parameter missing analyses under MNAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	-0.0019	0.0460	0.0447	0.942	-0.0013	0.0440	0.0447	0.956	0.0021	0.0465	0.0447	0.939
β_1	-0.1508	0.0392	0.0383	0.024	-0.2670	0.0429	0.0402	0.000	-0.5969	0.0670	0.0440	0.000
β_2	-0.1538	0.0390	0.0383	0.018	-0.2733	0.0440	0.0403	0.000	-0.5983	0.0692	0.0439	0.000
β_3	-0.1528	0.0407	0.0384	0.027	-0.2756	0.0455	0.0403	0.000	-0.5977	0.0756	0.0439	0.000
σ^2	-0.2825	0.0561	NA	NA	-0.4335	0.0549	NA	NA	-0.7055	0.0642	NA	NA
ρ	-0.1122	0.0178	NA	NA	-0.1794	0.0253	NA	NA	-0.3283	0.0413	NA	NA
EM												
β_0	-0.0019	0.0460	0.0447	0.942	-0.0013	0.0440	0.0447	0.956	0.0021	0.0465	0.0447	0.939
β_1	-0.0784	0.0366	0.0344	0.388	-0.1508	0.0371	0.0345	0.012	-0.3809	0.0407	0.0351	0.000
β_2	-0.0789	0.0357	0.0344	0.373	-0.1528	0.0374	0.0345	0.013	-0.3800	0.0410	0.0351	0.000
β_3	-0.0774	0.0359	0.0345	0.379	-0.1530	0.0359	0.0345	0.009	-0.3818	0.0405	0.0351	0.000
σ^2	-0.2250	0.0607	NA	NA	-0.3636	0.0570	NA	NA	-0.6280	0.0550	NA	NA
ρ	-0.0242	0.0136	NA	NA	-0.0435	0.0143	NA	NA	-0.0947	0.0180	NA	NA

Table 9.1. Continue

LMM												
β_0	-0.0019	0.0460	0.0423	0.933	-0.0013	0.0440	0.0409	0.936	0.0021	0.0465	0.0386	0.899
β_1	-0.0744	0.0358	0.0345	0.433	-0.1391	0.0355	0.0348	0.022	-0.3278	0.0379	0.0365	0.000
β_2	-0.0749	0.0353	0.0345	0.426	-0.1407	0.0361	0.0348	0.023	-0.3270	0.0385	0.0365	0.000
β_3	-0.0735	0.0353	0.0345	0.432	-0.1409	0.0347	0.0348	0.016	-0.3287	0.0380	0.0365	0.000
σ^2	-0.2114	0.0611	NA	NA	-0.3297	0.0599	NA	NA	-0.5070	0.0623	NA	NA
ρ	-0.3730	0.0262	NA	NA	-0.4007	0.0249	NA	NA	-0.4728	0.0553	NA	NA
GEE												
β_0	-0.0019	0.0460	0.0447	0.942	-0.0013	0.0440	0.0447	0.956	0.0021	0.0465	0.0447	0.939
β_1	-0.0780	0.0359	0.0350	0.405	-0.1479	0.0356	0.0356	0.013	-0.3525	0.0376	0.0376	0.000
β_2	-0.0785	0.0354	0.0350	0.395	-0.1495	0.0362	0.0356	0.015	-0.3518	0.0382	0.0376	0.000
β_3	-0.0770	0.0354	0.0350	0.395	-0.1497	0.0349	0.0356	0.010	-0.3534	0.0379	0.0376	0.000
σ^2	-0.2766	0.0559	NA	NA	-0.4120	0.0527	NA	NA	-0.6051	0.0539	NA	NA
ρ	-0.0680	0.0140	NA	NA	-0.0971	0.0153	NA	NA	-0.1339	0.0202	NA	NA
PROPOSED												
β_0	-0.0019	0.0460	0.0407	0.919	-0.0013	0.0440	0.0388	0.917	0.0021	0.0465	0.0360	0.880
β_1	-0.0776	0.0359	0.0353	0.418	-0.1480	0.0356	0.0359	0.014	-0.3641	0.0383	0.0387	0.000
β_2	-0.0782	0.0354	0.0353	0.410	-0.1496	0.0362	0.0359	0.014	-0.3633	0.0389	0.0387	0.000
β_3	-0.0767	0.0354	0.0353	0.404	-0.1498	0.0349	0.0359	0.011	-0.3650	0.0386	0.0387	0.000
σ^2	-0.3452	0.0527	NA	NA	-0.4917	0.0502	NA	NA	-0.7006	0.0485	NA	NA
ρ	-0.0636	0.0146	NA	NA	-0.0977	0.0165	NA	NA	-0.1673	0.0212	NA	NA

Table 9.2. Results of four parameter missing analyses under MNAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: AR(1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	0.0000	0.0455	0.0447	0.943	0.0007	0.0436	0.0447	0.952	-0.0019	0.0446	0.0447	0.953
β_1	-0.1524	0.0381	0.0382	0.017	-0.2705	0.0439	0.0403	0.000	-0.5945	0.0706	0.0439	0.000
β_2	-0.1556	0.0478	0.0459	0.079	-0.2761	0.0492	0.0465	0.000	-0.5981	0.0701	0.0477	0.000
β_3	-0.1532	0.0524	0.0506	0.157	-0.2786	0.0549	0.0505	0.000	-0.6007	0.0804	0.0501	0.000
σ^2	-0.2851	0.0527	NA	NA	-0.4325	0.0497	NA	NA	-0.7020	0.0636	NA	NA
ρ	-0.0821	0.0175	NA	NA	-0.1403	0.0249	NA	NA	-0.2869	0.0452	NA	NA
EM												
β_0	0.0000	0.0455	0.0447	0.943	0.0007	0.0436	0.0447	0.952	-0.0019	0.0446	0.0447	0.953
β_1	-0.0741	0.0349	0.0343	0.413	-0.1444	0.0339	0.0343	0.012	-0.3681	0.0404	0.0347	0.000
β_2	-0.0915	0.0455	0.0441	0.457	-0.1792	0.0442	0.0438	0.016	-0.4469	0.0473	0.0434	0.000
β_3	-0.1084	0.0501	0.0498	0.420	-0.2126	0.0493	0.0492	0.008	-0.5063	0.0508	0.0481	0.000
σ^2	-0.2456	0.0559	NA	NA	-0.3915	0.0495	NA	NA	-0.6638	0.0486	NA	NA
ρ	-0.0237	0.0143	NA	NA	-0.0449	0.0159	NA	NA	-0.0992	0.0213	NA	NA
LMM												
β_0	-0.0051	0.0454	0.0418	0.921	-0.0056	0.0536	0.0402	0.929	-0.0044	0.0445	0.0377	0.902
β_1	-0.0791	0.0344	0.0344	0.363	-0.1487	0.0332	0.0346	0.007	-0.3488	0.0379	0.0365	0.000
β_2	-0.1055	0.0455	0.0437	0.346	-0.2028	0.0432	0.0433	0.001	-0.4739	0.0460	0.0437	0.000
β_3	-0.1216	0.0498	0.0491	0.317	-0.2353	0.0489	0.0483	0.001	-0.5415	0.0504	0.0481	0.000
σ^2	-0.2410	0.0550	NA	NA	-0.3659	0.0501	NA	NA	-0.5582	0.0548	NA	NA
ρ	-0.0426	0.0227	NA	NA	-0.0619	0.0239	NA	NA	-0.1022	0.0269	NA	NA
GEE												
β_0	-0.0063	0.0453	0.0446	0.938	-0.0065	0.0437	0.0447	0.953	-0.0096	0.0444	0.0448	0.948
β_1	-0.0805	0.0344	0.0349	0.356	-0.1524	0.0333	0.0354	0.006	-0.3628	0.0380	0.0373	0.000
β_2	-0.1071	0.0455	0.0448	0.355	-0.2066	0.0433	0.0451	0.001	-0.4868	0.0458	0.0463	0.000
β_3	-0.1238	0.0499	0.0505	0.320	-0.2396	0.0489	0.0505	0.001	-0.5533	0.0502	0.0511	0.000
σ^2	-0.2818	0.0526	NA	NA	-0.4196	0.0468	NA	NA	-0.6294	0.0487	NA	NA
ρ	-0.0560	0.0149	NA	NA	-0.0854	0.0167	NA	NA	-0.1436	0.0236	NA	NA
PROPOSED												
β_0	0.0000	0.0455	0.0412	0.919	0.0007	0.0436	0.0395	0.930	-0.0019	0.0446	0.0368	0.894
β_1	-0.0724	0.0343	0.0350	0.455	-0.1377	0.0332	0.0354	0.024	-0.3359	0.0380	0.0376	0.000
β_2	-0.0916	0.0452	0.0447	0.465	-0.1784	0.0433	0.0447	0.018	-0.4374	0.0456	0.0461	0.000
β_3	-0.1092	0.0498	0.0502	0.425	-0.2125	0.0484	0.0498	0.008	-0.5042	0.0507	0.0506	0.000
σ^2	-0.3009	0.0517	NA	NA	-0.4374	0.0467	NA	NA	-0.6451	0.0480	NA	NA
ρ	-0.0483	0.0128	NA	NA	-0.0738	0.0144	NA	NA	-0.1248	0.0191	NA	NA

Table 9.3. Results of four parameter missing analyses under MNAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: Exchangeable

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	-0.0011	0.0453	0.0447	0.949	0.0046	0.0442	0.0447	0.953	0.0003	0.0439	0.0447	0.960
β_1	-0.1473	0.0540	0.0548	0.248	-0.2735	0.0567	0.0539	0.000	-0.5977	0.0826	0.0524	0.000
β_2	-0.1505	0.0550	0.0548	0.210	-0.2772	0.0586	0.0539	0.002	-0.6007	0.0827	0.0524	0.000
β_3	-0.1514	0.0574	0.0548	0.221	-0.2813	0.0585	0.0539	0.001	-0.6037	0.0853	0.0524	0.000
σ^2	-0.2865	0.0408	NA	NA	-0.4341	0.0417	NA	NA	-0.7021	0.0529	NA	NA
ρ	-0.0398	0.0153	NA	NA	-0.0625	0.0162	NA	NA	-0.1084	0.0171	NA	NA
EM												
β_0	-0.0011	0.0453	0.0447	0.949	0.0046	0.0442	0.0447	0.953	0.0003	0.0439	0.0447	0.960
β_1	-0.1406	0.0556	0.0544	0.296	-0.2637	0.0561	0.0532	0.002	-0.5811	0.0574	0.0513	0.000
β_2	-0.1429	0.0554	0.0544	0.257	-0.2636	0.0570	0.0533	0.001	-0.5811	0.0577	0.0513	0.000
β_3	-0.1417	0.0560	0.0544	0.270	-0.2676	0.0553	0.0533	0.002	-0.5793	0.0548	0.0513	0.000
σ^2	-0.2879	0.0397	NA	NA	-0.4361	0.0385	NA	NA	-0.7000	0.0362	NA	NA
ρ	-0.0263	0.0164	NA	NA	-0.0396	0.0181	NA	NA	-0.0684	0.0193	NA	NA
LMM												
β_0	-0.0011	0.0453	0.0414	0.933	0.0046	0.0442	0.0397	0.929	0.0003	0.0439	0.0368	0.910
β_1	-0.1398	0.0547	0.0540	0.295	-0.2625	0.0542	0.0530	0.001	-0.5771	0.0551	0.0527	0.000
β_2	-0.1417	0.0546	0.0540	0.244	-0.2625	0.0562	0.0530	0.001	-0.5775	0.0541	0.0527	0.000
β_3	-0.1416	0.0555	0.0540	0.272	-0.2657	0.0538	0.0530	0.001	-0.5755	0.0522	0.0527	0.000
σ^2	-0.2823	0.0394	NA	NA	-0.4221	0.0379	NA	NA	-0.6432	0.0381	NA	NA
ρ	-0.5494	0.0225	NA	NA	-0.5648	0.0257	NA	NA	-0.5976	0.0207	NA	NA
GEE												
β_0	-0.0011	0.0453	0.0447	0.949	0.0046	0.0442	0.0447	0.953	0.0003	0.0439	0.0447	0.960
β_1	-0.1399	0.0547	0.0552	0.307	-0.2627	0.0542	0.0546	0.001	-0.5776	0.0551	0.0545	0.000
β_2	-0.1418	0.0546	0.0552	0.265	-0.2626	0.0562	0.0547	0.002	-0.5780	0.0542	0.0545	0.000
β_3	-0.1417	0.0555	0.0552	0.283	-0.2658	0.0538	0.0547	0.001	-0.5760	0.0522	0.0545	0.000
σ^2	-0.2848	0.0393	NA	NA	-0.4245	0.0379	NA	NA	-0.6456	0.0380	NA	NA
ρ	-0.0278	0.0159	NA	NA	-0.0407	0.0174	NA	NA	-0.0669	0.0183	NA	NA
PROPOSED												
β_0	-0.0011	0.0453	0.0414	0.933	0.0046	0.0442	0.0396	0.928	0.0003	0.0439	0.0368	0.910
β_1	-0.1402	0.0547	0.0542	0.296	-0.2634	0.0542	0.0533	0.001	-0.5793	0.0554	0.0530	0.000
β_2	-0.1421	0.0546	0.0542	0.244	-0.2633	0.0562	0.0533	0.001	-0.5797	0.0544	0.0530	0.000
β_3	-0.1420	0.0556	0.0542	0.272	-0.2665	0.0538	0.0533	0.001	-0.5777	0.0523	0.0530	0.000
σ^2	-0.2876	0.0391	NA	NA	-0.4279	0.0379	NA	NA	-0.6478	0.0379	NA	NA
ρ	-0.0338	0.0166	NA	NA	-0.0496	0.0190	NA	NA	-0.0780	0.0224	NA	NA

Table 9.4. Results of four parameter missing analyses under MNAR

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_3(time$_{(4-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: AR(1)

Missing rate	5%				10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI												
β_0	0.0000	0.0552	0.0541	0.952	-0.0027	0.0426	0.0447	0.952	0.0000	0.0443	0.0447	0.948
β_1	-0.1517	0.0546	0.0537	0.211	-0.2695	0.0578	0.0539	0.000	-0.5991	0.0799	0.0523	0.000
β_2	-0.1536	0.0599	0.0582	0.265	-0.2733	0.0616	0.0576	0.005	-0.5993	0.0866	0.0547	0.000
β_3	-0.1545	0.0630	0.0584	0.279	-0.2731	0.0654	0.0583	0.010	-0.5992	0.0927	0.0552	0.000
σ^2	-0.2859	0.0672	NA	NA	-0.4325	0.0405	NA	NA	-0.7059	0.0573	NA	NA
ρ	-0.0420	0.0546	NA	NA	-0.0680	0.0181	NA	NA	-0.1173	0.0193	NA	NA
EM												
β_0	0.0000	0.0552	0.0541	0.952	-0.0027	0.0426	0.0447	0.952	0.0000	0.0443	0.0447	0.948
β_1	-0.1461	0.0551	0.0533	0.236	-0.2616	0.0552	0.0533	0.002	-0.5833	0.0578	0.0513	0.000
β_2	-0.1470	0.0600	0.0582	0.306	-0.2644	0.0590	0.0575	0.004	-0.5889	0.0594	0.0546	0.000
β_3	-0.1501	0.0622	0.0586	0.301	-0.2687	0.0600	0.0583	0.008	-0.5941	0.0612	0.0552	0.000
σ^2	-0.2875	0.0669	NA	NA	-0.4336	0.0378	NA	NA	-0.7005	0.0362	NA	NA
ρ	-0.0284	0.0545	NA	NA	-0.0457	0.0205	NA	NA	-0.0786	0.0236	NA	NA
LMM												
β_0	-0.0028	0.0551	0.0405	0.926	-0.0073	0.0530	0.0398	0.925	-0.0037	0.0443	0.0368	0.890
β_1	-0.1458	0.0544	0.0541	0.231	-0.2610	0.0543	0.0531	0.001	-0.5807	0.0557	0.0530	0.000
β_2	-0.1481	0.0594	0.0571	0.288	-0.2669	0.0575	0.0564	0.005	-0.5927	0.0579	0.0550	0.000
β_3	-0.1503	0.0616	0.0575	0.286	-0.2685	0.0586	0.0572	0.004	-0.5956	0.0596	0.0556	0.000
σ^2	-0.2806	0.0669	NA	NA	-0.4187	0.0373	NA	NA	-0.6428	0.0372	NA	NA
ρ	-0.0334	0.0558	NA	NA	-0.0516	0.0237	NA	NA	-0.0858	0.0274	NA	NA

Table 9.4. Continue

GEE												
β_0	-0.0029	0.0552	0.0540	0.950	-0.0065	0.0427	0.0447	0.948	-0.0038	0.0443	0.0447	0.944
β_1	-0.1460	0.0547	0.0542	0.249	-0.2614	0.0542	0.0547	0.001	-0.5826	0.0557	0.0545	0.000
β_2	-0.1482	0.0596	0.0588	0.304	-0.2672	0.0575	0.0588	0.006	-0.5937	0.0578	0.0577	0.000
β_3	-0.1505	0.0616	0.0593	0.307	-0.2686	0.0588	0.0596	0.006	-0.5962	0.0596	0.0583	0.000
σ^2	-0.2838	0.0667	NA	NA	-0.4222	0.0371	NA	NA	-0.6462	0.0370	NA	NA
ρ	-0.0315	0.0544	NA	NA	-0.0509	0.0187	NA	NA	-0.0893	0.0203	NA	NA
PROPOSED												
β_0	0.0000	0.0552	0.0514	0.927	-0.0027	0.0426	0.0398	0.925	0.0000	0.0443	0.0368	0.887
β_1	-0.1454	0.0548	0.0530	0.232	-0.2603	0.0542	0.0531	0.001	-0.5788	0.0557	0.0528	0.000
β_2	-0.1466	0.0596	0.0575	0.301	-0.2642	0.0574	0.0570	0.005	-0.5875	0.0579	0.0557	0.000
β_3	-0.1500	0.0616	0.0579	0.289	-0.2676	0.0587	0.0576	0.006	-0.5930	0.0596	0.0562	0.000
σ^2	-0.2809	0.0668	NA	NA	-0.4191	0.0374	NA	NA	-0.6429	0.0374	NA	NA
ρ	-0.0245	0.0546	NA	NA	-0.0392	0.0210	NA	NA	-0.0640	0.0284	NA	NA

Table 10.1. Results of four parameter missing analyses under MCAR with missing rates of (time1, time2, time3) in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	(10%, 25%, 50%), (10%, 25%, 50%)				(5%, 10%, 25%), (10%, 25%, 50%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC								
β_0	-0.0022	0.0992	0.0978	0.939	-0.0032	0.0717	0.0742	0.956
β_1	-0.0013	0.0554	0.0565	0.956	-0.0011	0.0479	0.0478	0.956
β_2	-0.0008	0.0555	0.0565	0.952	-0.0004	0.0489	0.0477	0.943
β_3	0.0067	0.1326	0.1305	0.945	0.0057	0.1138	0.1150	0.955
σ^2	-0.0062	0.1195	NA	NA	-0.0045	0.0987	NA	NA
ρ	-0.0010	0.0218	NA	NA	-0.0007	0.0181	NA	NA
LOCF								
β_0	-0.0005	0.0600	0.0609	0.954	-0.0028	0.0594	0.0603	0.955
β_1	0.0001	0.0296	0.0300	0.955	-0.0010	0.0312	0.0314	0.961
β_2	-0.0005	0.0319	0.0324	0.949	-0.0010	0.0337	0.0333	0.949
β_3	0.0036	0.0796	0.0828	0.966	0.0032	0.0816	0.0820	0.951
σ^2	-0.0042	0.0754	NA	NA	-0.0017	0.0771	NA	NA
ρ	0.0872	0.0106	NA	NA	0.0621	0.0116	NA	NA
MI								
β_0	-0.0011	0.0668	0.0563	0.907	-0.0024	0.0648	0.0589	0.927
β_1	-0.0041	0.0599	0.0435	0.882	-0.0035	0.0506	0.0411	0.897
β_2	-0.0063	0.1338	0.0505	0.662	-0.0023	0.1038	0.0473	0.701
β_3	0.0047	0.0969	0.0683	0.854	0.0023	0.0964	0.0715	0.863
σ^2	-0.0181	0.1220	NA	NA	-0.0083	0.1064	NA	NA
ρ	-0.3159	0.0223	NA	NA	-0.2373	0.0211	NA	NA
EM								
β_0	-0.0008	0.0593	0.0599	0.948	-0.0029	0.0588	0.0600	0.956
β_1	-0.0015	0.0408	0.0346	0.899	-0.0015	0.0384	0.0346	0.922
β_2	-0.0013	0.0505	0.0346	0.812	-0.0012	0.0461	0.0346	0.865
β_3	0.0041	0.0777	0.0799	0.955	0.0032	0.0790	0.0800	0.953
σ^2	-0.0058	0.0796	NA	NA	-0.0014	0.0791	NA	NA
ρ	-0.0005	0.0166	NA	NA	-0.0005	0.0156	NA	NA
LMM								
β_0	-0.0004	0.0598	0.0608	0.954	-0.0027	0.0596	0.0606	0.952
β_1	-0.0006	0.0382	0.0389	0.954	-0.0018	0.0369	0.0374	0.955
β_2	-0.0012	0.0453	0.0457	0.947	-0.0009	0.0422	0.0416	0.942
β_3	0.0032	0.0787	0.0823	0.967	0.0030	0.0809	0.0817	0.953
σ^2	-0.0003	0.0748	NA	NA	0.0016	0.0765	NA	NA
ρ	-0.2652	0.0177	NA	NA	-0.2668	0.0162	NA	NA
GEE								
β_0	-0.0004	0.0598	0.0607	0.953	-0.0027	0.0596	0.0605	0.953
β_1	-0.0006	0.0382	0.0388	0.953	-0.0018	0.0370	0.0373	0.954
β_2	-0.0012	0.0453	0.0456	0.947	-0.0009	0.0422	0.0416	0.946
β_3	0.0032	0.0787	0.0822	0.966	0.0030	0.0809	0.0816	0.953
σ^2	-0.0035	0.0765	NA	NA	-0.0021	0.0772	NA	NA
ρ	0.0004	0.0226	NA	NA	-0.0003	0.0192	NA	NA
PROPOSED								
β_0	-0.0004	0.0598	0.0608	0.954	-0.0027	0.0596	0.0606	0.955
β_1	-0.0006	0.0382	0.0389	0.955	-0.0018	0.0369	0.0374	0.954
β_2	-0.0012	0.0453	0.0457	0.948	-0.0009	0.0422	0.0416	0.943
β_3	0.0032	0.0787	0.0823	0.965	0.0030	0.0809	0.0817	0.952
σ^2	0.0025	0.0910	NA	NA	0.0033	0.0844	NA	NA
ρ	-0.0003	0.0218	NA	NA	-0.0002	0.0180	NA	NA

Table 10.2. Results of four parameter missing analyses under MCAR with missing rates of (time1, time2, time3) in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: AR(1)

Missing rate	(10%, 25%, 50%), (10%, 25%, 50%)				(5%, 10%, 25%), (10%, 25%, 50%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC								
β_0	-0.0022	0.1002	0.1014	0.954	0.0014	0.0736	0.0751	0.954
β_1	-0.0016	0.0604	0.0596	0.945	-0.0022	0.0474	0.0494	0.953
β_2	-0.0012	0.0793	0.0776	0.951	-0.0021	0.0634	0.0644	0.947
β_3	-0.0008	0.1335	0.1320	0.939	-0.0013	0.1152	0.1151	0.946
σ^2	-0.0128	0.1227	NA	NA	-0.0140	0.0966	NA	NA
ρ	-0.0020	0.0251	NA	NA	-0.0019	0.0200	NA	NA
LOCF								
β_0	-0.0033	0.0593	0.0614	0.951	0.0014	0.0580	0.0597	0.960
β_1	-0.0007	0.0312	0.0315	0.960	0.0001	0.0313	0.0326	0.950
β_2	-0.0002	0.0399	0.0395	0.955	-0.0009	0.0409	0.0413	0.947
β_3	0.0017	0.0785	0.0816	0.957	-0.0015	0.0804	0.0801	0.944
σ^2	-0.0041	0.0787	NA	NA	-0.0098	0.0730	NA	NA
ρ	0.0976	0.0125	NA	NA	0.0695	0.0127	NA	NA
MI								
β_0	-0.0044	0.0720	0.0555	0.888	0.0022	0.0621	0.0577	0.929
β_1	0.0000	0.0491	0.0459	0.934	-0.0004	0.0426	0.0430	0.941
β_2	0.0028	0.1262	0.0555	0.691	-0.0003	0.0956	0.0532	0.781
β_3	0.0015	0.1039	0.0658	0.817	-0.0031	0.0990	0.0687	0.842
σ^2	-0.0148	0.1426	NA	NA	-0.0180	0.1065	NA	NA
ρ	-0.3033	0.0260	NA	NA	-0.2197	0.0219	NA	NA
EM								
β_0	-0.0029	0.0589	0.0589	0.946	0.0017	0.0591	0.0589	0.953
β_1	-0.0009	0.0433	0.0346	0.876	-0.0003	0.0392	0.0346	0.914
β_2	-0.0007	0.0651	0.0450	0.834	-0.0023	0.0564	0.0451	0.874
β_3	0.0006	0.0741	0.0767	0.953	-0.0021	0.0777	0.0767	0.946
σ^2	-0.0072	0.0798	NA	NA	-0.0089	0.0747	NA	NA
ρ	-0.0004	0.0194	NA	NA	-0.0008	0.0177	NA	NA
LMM								
β_0	-0.0033	0.0596	0.0614	0.953	0.0017	0.0588	0.0602	0.960
β_1	-0.0008	0.0399	0.0409	0.964	-0.0005	0.0370	0.0386	0.951
β_2	0.0002	0.0595	0.0553	0.944	-0.0021	0.0536	0.0514	0.933
β_3	0.0017	0.0782	0.0814	0.956	-0.0017	0.0801	0.0801	0.949
σ^2	-0.0017	0.0768	NA	NA	-0.0067	0.0713	NA	NA
ρ	-0.0601	0.0595	NA	NA	-0.0480	0.0505	NA	NA
GEE								
β_0	-0.0033	0.0597	0.0617	0.955	0.0017	0.0588	0.0606	0.961
β_1	-0.0008	0.0400	0.0404	0.960	-0.0006	0.0370	0.0385	0.953
β_2	0.0001	0.0596	0.0589	0.955	-0.0021	0.0537	0.0540	0.950
β_3	0.0016	0.0782	0.0812	0.955	-0.0017	0.0800	0.0799	0.950
σ^2	-0.0053	0.0783	NA	NA	-0.0105	0.0725	NA	NA
ρ	-0.0140	0.0251	NA	NA	-0.0098	0.0210	NA	NA
PROPOSED								
β_0	-0.0033	0.0594	0.0615	0.952	0.0015	0.0585	0.0604	0.958
β_1	-0.0008	0.0398	0.0402	0.964	-0.0003	0.0367	0.0382	0.953
β_2	0.0001	0.0584	0.0575	0.948	-0.0015	0.0526	0.0529	0.950
β_3	0.0016	0.0781	0.0812	0.957	-0.0016	0.0800	0.0799	0.951
σ^2	-0.0008	0.0891	NA	NA	-0.0062	0.0773	NA	NA
ρ	-0.0014	0.0227	NA	NA	-0.0014	0.0179	NA	NA

Table 10.3. Results of four parameter model under MCAR with missing rates of (time1, time2, time3) in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: Exchangeable

Missing rate	(10%, 25%, 50%), (10%, 25%, 50%)				(5%, 10%, 25%), (10%, 25%, 50%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC								
β_0	0.0032	0.0911	0.0883	0.937	0.0001	0.0671	0.0691	0.959
β_1	-0.0020	0.0926	0.0922	0.955	0.0004	0.0770	0.0780	0.950
β_2	-0.0017	0.0923	0.0922	0.943	-0.0035	0.0770	0.0779	0.951
β_3	-0.0020	0.1034	0.0995	0.951	-0.0020	0.0863	0.0878	0.954
σ^2	-0.0096	0.0882	NA	NA	-0.0063	0.0756	NA	NA
ρ	-0.0010	0.0325	NA	NA	-0.0009	0.0288	NA	NA
LOCF								
β_0	-0.0024	0.0555	0.0570	0.957	-0.0002	0.0538	0.0549	0.950
β_1	0.0012	0.0461	0.0489	0.967	0.0008	0.0518	0.0513	0.945
β_2	0.0016	0.0502	0.0528	0.960	-0.0011	0.0554	0.0543	0.952
β_3	0.0011	0.0715	0.0705	0.950	-0.0013	0.0659	0.0679	0.965
σ^2	-0.0040	0.0674	NA	NA	-0.0052	0.0609	NA	NA
ρ	0.2336	0.0210	NA	NA	0.1674	0.0221	NA	NA
MI								
β_0	-0.0029	0.0623	0.0530	0.907	0.0002	0.0596	0.0537	0.926
β_1	0.0014	0.0669	0.0582	0.926	0.0013	0.0645	0.0577	0.919
β_2	-0.0002	0.1441	0.0597	0.650	0.0020	0.1185	0.0589	0.706
β_3	0.0020	0.0888	0.0568	0.835	-0.0021	0.0784	0.0579	0.864
σ^2	-0.0167	0.1163	NA	NA	-0.0142	0.0921	NA	NA
ρ	-0.0904	0.0168	NA	NA	-0.0675	0.0189	NA	NA
EM								
β_0	-0.0021	0.0538	0.0541	0.963	-0.0007	0.0530	0.0541	0.950
β_1	0.0019	0.0604	0.0565	0.931	0.0004	0.0618	0.0564	0.924
β_2	0.0012	0.0757	0.0565	0.855	-0.0021	0.0699	0.0565	0.881
β_3	0.0004	0.0635	0.0610	0.948	-0.0002	0.0596	0.0610	0.955
σ^2	-0.0055	0.0692	NA	NA	-0.0061	0.0613	NA	NA
ρ	0.0002	0.0298	NA	NA	0.0003	0.0280	NA	NA
LMM								
β_0	-0.0021	0.0555	0.0560	0.963	0.0000	0.0542	0.0555	0.955
β_1	0.0020	0.0568	0.0618	0.967	0.0004	0.0606	0.0600	0.949
β_2	0.0023	0.0695	0.0712	0.959	-0.0025	0.0661	0.0658	0.934
β_3	0.0004	0.0696	0.0674	0.949	-0.0017	0.0635	0.0657	0.961
σ^2	-0.0009	0.0626	NA	NA	-0.0029	0.0576	NA	NA
ρ	-0.4036	0.0168	NA	NA	-0.4130	0.0163	NA	NA
GEE								
β_0	-0.0021	0.0555	0.0560	0.960	0.0000	0.0542	0.0554	0.957
β_1	0.0020	0.0568	0.0618	0.967	0.0004	0.0606	0.0599	0.949
β_2	0.0023	0.0695	0.0712	0.956	-0.0025	0.0661	0.0658	0.935
β_3	0.0004	0.0695	0.0673	0.949	-0.0017	0.0635	0.0656	0.960
σ^2	-0.0047	0.0625	NA	NA	-0.0064	0.0576	NA	NA
ρ	0.0001	0.0264	NA	NA	0.0002	0.0257	NA	NA
PROPOSED								
β_0	-0.0021	0.0554	0.0560	0.963	0.0000	0.0542	0.0555	0.955
β_1	0.0020	0.0568	0.0619	0.967	0.0004	0.0606	0.0600	0.950
β_2	0.0023	0.0695	0.0712	0.958	-0.0025	0.0661	0.0659	0.935
β_3	0.0004	0.0695	0.0674	0.949	-0.0017	0.0635	0.0657	0.959
σ^2	-0.0002	0.0635	NA	NA	-0.0028	0.0581	NA	NA
ρ	0.0000	0.0325	NA	NA	-0.0002	0.0288	NA	NA

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Table 10.4. Results of four parameter missing analyses under MCAR with missing rates of (time1, time2, time3) in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(time$_{(3\text{-}1)}$) = β_2(group$_{(2\text{-}1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: AR(1)

Missing rate	(10%, 25%, 50%), (10%, 25%, 50%)				(5%, 10%, 25%), (10%, 25%, 50%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
CC								
β_0	-0.0041	0.0922	0.0919	0.948	0.0036	0.0725	0.0708	0.943
β_1	0.0004	0.0999	0.0975	0.944	-0.0027	0.0799	0.0808	0.945
β_2	0.0040	0.1091	0.1065	0.945	-0.0016	0.0887	0.0886	0.948
β_3	0.0018	0.0967	0.1005	0.949	-0.0035	0.0877	0.0879	0.954
σ^2	-0.0092	0.0918	NA	NA	-0.0023	0.0740	NA	NA
ρ	-0.0028	0.0387	NA	NA	-0.0006	0.0317	NA	NA
LOCF								
β_0	-0.0012	0.0574	0.0584	0.956	0.0006	0.0532	0.0554	0.962
β_1	-0.0008	0.0510	0.0510	0.941	-0.0019	0.0521	0.0529	0.957
β_2	0.0002	0.0573	0.0576	0.949	-0.0017	0.0591	0.0593	0.952
β_3	-0.0006	0.0694	0.0698	0.949	-0.0005	0.0664	0.0667	0.957
σ^2	-0.0056	0.0677	NA	NA	-0.0050	0.0615	NA	NA
ρ	0.2860	0.0222	NA	NA	0.2084	0.0229	NA	NA
MI								
β_0	-0.0034	0.0681	0.0526	0.868	0.0018	0.0606	0.0531	0.909
β_1	-0.0031	0.0648	0.0588	0.925	-0.0025	0.0597	0.0582	0.947
β_2	-0.0031	0.1419	0.0625	0.664	-0.0009	0.1101	0.0624	0.758
β_3	0.0026	0.0920	0.0552	0.803	-0.0037	0.0885	0.0561	0.812
σ^2	-0.0098	0.1382	NA	NA	-0.0071	0.1086	NA	NA
ρ	-0.0935	0.0185	NA	NA	-0.0692	0.0193	NA	NA
EM								
β_0	-0.0006	0.0558	0.0534	0.942	0.0008	0.0543	0.0534	0.945
β_1	-0.0033	0.0689	0.0565	0.885	-0.0010	0.0639	0.0565	0.914
β_2	0.0019	0.0833	0.0618	0.862	0.0003	0.0774	0.0620	0.885
β_3	0.0001	0.0573	0.0584	0.947	-0.0017	0.0585	0.0585	0.948
σ^2	-0.0072	0.0682	NA	NA	-0.0025	0.0627	NA	NA
ρ	-0.0015	0.0346	NA	NA	0.0004	0.0298	NA	NA
LMM								
β_0	-0.0014	0.0565	0.0577	0.957	0.0011	0.0549	0.0561	0.958
β_1	-0.0018	0.0630	0.0640	0.952	-0.0016	0.0604	0.0616	0.955
β_2	0.0016	0.0766	0.0760	0.945	-0.0011	0.0716	0.0709	0.949
β_3	-0.0001	0.0660	0.0670	0.950	-0.0014	0.0648	0.0648	0.956
σ^2	-0.0024	0.0630	NA	NA	0.0000	0.0573	NA	NA
ρ	-0.0438	0.0493	NA	NA	-0.0317	0.0409	NA	NA
GEE								
β_0	-0.0013	0.0565	0.0577	0.958	0.0012	0.0549	0.0562	0.954
β_1	-0.0018	0.0630	0.0636	0.954	-0.0016	0.0604	0.0613	0.952
β_2	0.0015	0.0765	0.0775	0.947	-0.0011	0.0716	0.0720	0.949
β_3	-0.0001	0.0659	0.0668	0.949	-0.0015	0.0648	0.0645	0.956
σ^2	-0.0062	0.0628	NA	NA	-0.0034	0.0572	NA	NA
ρ	-0.0160	0.0297	NA	NA	-0.0101	0.0264	NA	NA
PROPOSED								
β_0	-0.0013	0.0563	0.0577	0.957	0.0011	0.0549	0.0562	0.957
β_1	-0.0018	0.0630	0.0636	0.951	-0.0016	0.0604	0.0613	0.953
β_2	0.0016	0.0763	0.0774	0.949	-0.0012	0.0715	0.0719	0.949
β_3	-0.0001	0.0658	0.0668	0.944	-0.0014	0.0647	0.0646	0.952
σ^2	-0.0016	0.0638	NA	NA	0.0007	0.0577	NA	NA
ρ	-0.0019	0.0372	NA	NA	0.0003	0.0305	NA	NA

Table 11.1. Results of four parameter missing analyses under MAR with missing rates in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0275	0.0577	0.0581	0.924	0.0716	0.0597	0.0573	0.745
β_1	-0.1470	0.0431	0.0422	0.057	-0.2894	0.0528	0.0463	0.000
β_2	-0.1496	0.0439	0.0402	0.046	-0.3096	0.0591	0.0440	0.000
β_3	-0.0566	0.0740	0.0713	0.861	-0.1423	0.0807	0.0658	0.455
σ^2	-0.1561	0.0658	NA	NA	-0.2415	0.0677	NA	NA
ρ	-0.1652	0.0218	NA	NA	-0.2756	0.0327	NA	NA
EM								
β_0	0.0021	0.0583	0.0598	0.951	0.0089	0.0602	0.0596	0.952
β_1	-0.0051	0.0376	0.0348	0.933	-0.0130	0.0418	0.0349	0.881
β_2	-0.0138	0.0373	0.0349	0.910	-0.0444	0.0432	0.0353	0.733
β_3	-0.0056	0.0788	0.0792	0.961	-0.0168	0.0804	0.0783	0.939
σ^2	-0.0288	0.0750	NA	NA	-0.0583	0.0771	NA	NA
ρ	-0.0073	0.0143	NA	NA	-0.0161	0.0170	NA	NA
LMM								
β_0	0.0019	0.0586	0.0598	0.950	0.0081	0.0608	0.0596	0.945
β_1	-0.0051	0.0355	0.0359	0.947	-0.0145	0.0373	0.0378	0.940
β_2	-0.0120	0.0364	0.0359	0.930	-0.0355	0.0405	0.0378	0.831
β_3	-0.0052	0.0795	0.0798	0.959	-0.0152	0.0826	0.0799	0.938
σ^2	-0.0227	0.0741	NA	NA	-0.0453	0.0738	NA	NA
ρ	-0.2744	0.0167	NA	NA	-0.2832	0.0225	NA	NA
GEE								
β_0	0.0048	0.0585	0.0597	0.945	0.0169	0.0606	0.0597	0.936
β_1	-0.0210	0.0360	0.0362	0.915	-0.0535	0.0384	0.0384	0.717
β_2	-0.0271	0.0367	0.0361	0.888	-0.0725	0.0409	0.0382	0.518
β_3	-0.0111	0.0786	0.0789	0.958	-0.0329	0.0817	0.0784	0.922
σ^2	-0.1431	0.0652	NA	NA	-0.1875	0.0648	NA	NA
ρ	-0.1019	0.0146	NA	NA	-0.1269	0.0182	NA	NA
PROPOSED								
β_0	0.0037	0.0585	0.0563	0.937	0.0156	0.0607	0.0550	0.914
β_1	-0.0153	0.0358	0.0374	0.943	-0.0476	0.0382	0.0404	0.794
β_2	-0.0217	0.0366	0.0374	0.917	-0.0669	0.0410	0.0404	0.629
β_3	-0.0090	0.0789	0.0742	0.931	-0.0302	0.0817	0.0723	0.896
σ^2	-0.2037	0.0622	NA	NA	-0.2791	0.0591	NA	NA
ρ	-0.0683	0.0158	NA	NA	-0.1100	0.0194	NA	NA

Table 11.2. Results of four parameter missing analyses under MAR with missing rates in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: AR(1)

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0267	0.0569	0.0572	0.929	0.0634	0.0585	0.0565	0.801
β_1	-0.1462	0.0431	0.0422	0.062	-0.2897	0.0549	0.0462	0.000
β_2	-0.1517	0.0522	0.0489	0.140	-0.3075	0.0662	0.0513	0.000
β_3	-0.0486	0.0719	0.0694	0.875	-0.1307	0.0756	0.0642	0.487
σ^2	-0.1571	0.0627	NA	NA	-0.2381	0.0712	NA	NA
ρ	-0.1446	0.0233	NA	NA	-0.2564	0.0350	NA	NA
EM								
β_0	0.0063	0.0575	0.0586	0.956	0.0118	0.0569	0.0584	0.944
β_1	-0.0099	0.0366	0.0349	0.930	-0.0270	0.0415	0.0351	0.839
β_2	-0.0256	0.0473	0.0458	0.908	-0.0695	0.0532	0.0465	0.656
β_3	-0.0077	0.0756	0.0756	0.958	-0.0275	0.0708	0.0742	0.941
σ^2	-0.0389	0.0726	NA	NA	-0.0751	0.0752	NA	NA
ρ	-0.0142	0.0162	NA	NA	-0.0300	0.0194	NA	NA

Table 11.2. Continue

LMM								
β_0	0.0222	0.0581	0.0581	0.939	0.0374	0.0576	0.0576	0.892
β_1	-0.0351	0.0359	0.0364	0.846	-0.0702	0.0392	0.0386	0.547
β_2	-0.0749	0.0476	0.0463	0.635	-0.1470	0.0539	0.0482	0.166
β_3	-0.0121	0.0759	0.0758	0.951	-0.0354	0.0722	0.0756	0.928
σ^2	-0.0540	0.0695	NA	NA	-0.0842	0.0705	NA	NA
ρ	-0.0427	0.0327	NA	NA	-0.0574	0.0336	NA	NA
GEE								
β_0	0.0215	0.0577	0.0588	0.942	0.0385	0.0572	0.0588	0.899
β_1	-0.0425	0.0362	0.0367	0.791	-0.0917	0.0396	0.0389	0.354
β_2	-0.0884	0.0477	0.0488	0.555	-0.1799	0.0533	0.0515	0.058
β_3	-0.0179	0.0751	0.0748	0.948	-0.0515	0.0714	0.0741	0.891
σ^2	-0.1473	0.0619	NA	NA	-0.1980	0.0623	NA	NA
ρ	-0.0979	0.0164	NA	NA	-0.1352	0.0202	NA	NA
PROPOSED								
β_0	0.0076	0.0579	0.0560	0.948	0.0172	0.0575	0.0551	0.918
β_1	-0.0167	0.0356	0.0372	0.935	-0.0489	0.0387	0.0399	0.786
β_2	-0.0351	0.0463	0.0475	0.890	-0.0933	0.0517	0.0501	0.572
β_3	-0.0105	0.0761	0.0722	0.943	-0.0382	0.0721	0.0709	0.898
σ^2	-0.1642	0.0619	NA	NA	-0.2246	0.0605	NA	NA
ρ	-0.0566	0.0154	NA	NA	-0.0902	0.0180	NA	NA

Table 11.3. Results of four parameter missing analyses under MAR with missing analyses in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0119	0.0543	0.0537	0.938	0.0190	0.0553	0.0535	0.926
β_1	-0.0459	0.0582	0.0584	0.874	-0.0822	0.0612	0.0593	0.714
β_2	-0.0454	0.0583	0.0572	0.875	-0.0880	0.0740	0.0578	0.632
β_3	-0.0169	0.0621	0.0587	0.928	-0.0405	0.0663	0.0572	0.849
σ^2	-0.0128	0.0594	NA	NA	-0.0263	0.0663	NA	NA
ρ	-0.0489	0.0229	NA	NA	-0.0788	0.0255	NA	NA
EM								
β_0	0.0050	0.0538	0.0541	0.950	0.0030	0.0535	0.0540	0.950
β_1	-0.0055	0.0595	0.0566	0.938	-0.0031	0.0643	0.0566	0.915
β_2	-0.0072	0.0558	0.0567	0.952	-0.0202	0.0649	0.0569	0.894
β_3	-0.0032	0.0613	0.0609	0.951	-0.0085	0.0605	0.0606	0.942
σ^2	-0.0024	0.0599	NA	NA	-0.0116	0.0584	NA	NA
ρ	-0.0033	0.0240	NA	NA	-0.0086	0.0269	NA	NA
LMM								
β_0	0.0051	0.0542	0.0544	0.947	0.0039	0.0540	0.0547	0.948
β_1	-0.0059	0.0578	0.0580	0.947	-0.0071	0.0607	0.0602	0.941
β_2	-0.0074	0.0544	0.0580	0.962	-0.0197	0.0610	0.0602	0.939
β_3	-0.0034	0.0619	0.0621	0.954	-0.0104	0.0629	0.0634	0.937
σ^2	0.0004	0.0591	NA	NA	-0.0090	0.0557	NA	NA
ρ	-0.4184	0.0156	NA	NA	-0.4149	0.0149	NA	NA
GEE								
β_0	0.0060	0.0541	0.0544	0.948	0.0059	0.0540	0.0548	0.946
β_1	-0.0111	0.0579	0.0581	0.944	-0.0172	0.0608	0.0603	0.936
β_2	-0.0121	0.0544	0.0580	0.961	-0.0281	0.0609	0.0603	0.921
β_3	-0.0051	0.0617	0.0618	0.951	-0.0144	0.0628	0.0631	0.938
σ^2	-0.0118	0.0580	NA	NA	-0.0207	0.0549	NA	NA
ρ	-0.0308	0.0196	NA	NA	-0.0366	0.0209	NA	NA
PROPOSED								
β_0	0.0061	0.0541	0.0538	0.947	0.0073	0.0539	0.0539	0.941
β_1	-0.0118	0.0578	0.0588	0.946	-0.0241	0.0607	0.0614	0.930
β_2	-0.0127	0.0544	0.0588	0.960	-0.0338	0.0612	0.0614	0.917
β_3	-0.0053	0.0617	0.0605	0.947	-0.0171	0.0625	0.0612	0.929
σ^2	-0.0175	0.0572	NA	NA	-0.0304	0.0545	NA	NA
ρ	-0.0341	0.0217	NA	NA	-0.0541	0.0234	NA	NA

Table 11.4. Results of four parameter missing analyses under MAR with missing analyses in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(time$_{(3\text{-}1)}$) = β_2(group$_{(2\text{-}1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: AR(1)

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0083	0.0556	0.0530	0.926	0.0169	0.0546	0.0528	0.933
β_1	-0.0417	0.0608	0.0584	0.884	-0.0790	0.0635	0.0594	0.716
β_2	-0.0443	0.0688	0.0621	0.863	-0.0837	0.0779	0.0623	0.686
β_3	-0.0149	0.0579	0.0564	0.934	-0.0350	0.0640	0.0550	0.853
σ^2	-0.0165	0.0561	NA	NA	-0.0225	0.0676	NA	NA
ρ	-0.0598	0.0256	NA	NA	-0.0972	0.0314	NA	NA
EM								
β_0	0.0018	0.0552	0.0534	0.939	0.0023	0.0531	0.0534	0.951
β_1	-0.0014	0.0617	0.0566	0.929	-0.0035	0.0662	0.0568	0.905
β_2	-0.0072	0.0683	0.0621	0.924	-0.0205	0.0693	0.0625	0.908
β_3	-0.0021	0.0569	0.0583	0.955	-0.0058	0.0556	0.0579	0.950
σ^2	-0.0059	0.0561	NA	NA	-0.0062	0.0589	NA	NA
ρ	-0.0027	0.0280	NA	NA	-0.0122	0.0315	NA	NA
LMM								
β_0	0.0081	0.0555	0.0534	0.932	0.0127	0.0536	0.0537	0.947
β_1	-0.0133	0.0601	0.0586	0.938	-0.0290	0.0623	0.0614	0.927
β_2	-0.0395	0.0677	0.0626	0.873	-0.0720	0.0673	0.0646	0.788
β_3	-0.0079	0.0573	0.0590	0.953	-0.0186	0.0572	0.0603	0.955
σ^2	-0.0093	0.0536	NA	NA	-0.0112	0.0553	NA	NA
ρ	-0.0420	0.0347	NA	NA	-0.0637	0.0380	NA	NA
GEE								
β_0	0.0083	0.0554	0.0537	0.931	0.0135	0.0535	0.0540	0.948
β_1	-0.0160	0.0602	0.0583	0.940	-0.0357	0.0623	0.0609	0.914
β_2	-0.0427	0.0677	0.0642	0.877	-0.0780	0.0665	0.0664	0.780
β_3	-0.0093	0.0571	0.0586	0.953	-0.0218	0.0571	0.0597	0.943
σ^2	-0.0168	0.0531	NA	NA	-0.0187	0.0549	NA	NA
ρ	-0.0454	0.0219	NA	NA	-0.0694	0.0239	NA	NA
PROPOSED								
β_0	0.0029	0.0555	0.0533	0.933	0.0058	0.0536	0.0536	0.947
β_1	-0.0070	0.0599	0.0587	0.942	-0.0200	0.0621	0.0614	0.940
β_2	-0.0128	0.0668	0.0633	0.928	-0.0326	0.0660	0.0655	0.916
β_3	-0.0043	0.0579	0.0585	0.949	-0.0127	0.0579	0.0596	0.960
σ^2	-0.0156	0.0532	NA	NA	-0.0172	0.0550	NA	NA
ρ	-0.0291	0.0240	NA	NA	-0.0472	0.0278	NA	NA

Table 12.1. Results of four parameter missing analyses under MNAR with missing rates in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(time$_{(3\text{-}1)}$) = β_2(group$_{(2\text{-}1)}$) = 0, σ^2 = 2, ρ = 0.7 Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0421	0.0578	0.0573	0.880	0.0988	0.0588	0.0563	0.574
β_1	-0.2117	0.0414	0.0393	0.001	-0.4204	0.0501	0.0423	0.000
β_2	-0.2174	0.0438	0.0395	0.000	-0.4402	0.0526	0.0427	0.000
β_3	-0.0813	0.0722	0.0688	0.764	-0.1977	0.0737	0.0618	0.143
σ^2	-0.3182	0.0600	NA	NA	-0.4908	0.0639	NA	NA
ρ	-0.1449	0.0218	NA	NA	-0.2519	0.0307	NA	NA
EM								
β_0	0.0243	0.0583	0.0582	0.932	0.0542	0.0570	0.0571	0.841
β_1	-0.1181	0.0364	0.0345	0.084	-0.2573	0.0395	0.0348	0.000
β_2	-0.1189	0.0371	0.0345	0.080	-0.2554	0.0378	0.0347	0.000
β_3	-0.0458	0.0743	0.0736	0.899	-0.1085	0.0701	0.0694	0.656
σ^2	-0.2615	0.0642	NA	NA	-0.4157	0.0641	NA	NA
ρ	-0.0330	0.0156	NA	NA	-0.0604	0.0188	NA	NA

Table 12.1. Continue

LMM								
β_0	0.0229	0.0586	0.0562	0.923	0.0497	0.0577	0.0545	0.835
β_1	-0.1099	0.0353	0.0350	0.125	-0.2290	0.0376	0.0363	0.000
β_2	-0.1105	0.0367	0.0350	0.119	-0.2278	0.0359	0.0363	0.000
β_3	-0.0430	0.0753	0.0747	0.907	-0.0995	0.0731	0.0727	0.717
σ^2	-0.2371	0.0657	NA	NA	-0.3478	0.0688	NA	NA
ρ	-0.3050	0.0258	NA	NA	-0.3351	0.0364	NA	NA
GEE								
β_0	0.0244	0.0586	0.0584	0.928	0.0548	0.0576	0.0579	0.850
β_1	-0.1174	0.0355	0.0354	0.097	-0.2500	0.0375	0.0368	0.000
β_2	-0.1180	0.0369	0.0354	0.094	-0.2487	0.0361	0.0368	0.000
β_3	-0.0459	0.0750	0.0743	0.894	-0.1096	0.0724	0.0717	0.663
σ^2	-0.3055	0.0597	NA	NA	-0.4341	0.0604	NA	NA
ρ	-0.0857	0.0160	NA	NA	-0.1223	0.0189	NA	NA
PROPOSED								
β_0	0.0239	0.0586	0.0541	0.908	0.0540	0.0577	0.0516	0.792
β_1	-0.1151	0.0355	0.0360	0.112	-0.2467	0.0378	0.0379	0.000
β_2	-0.1158	0.0368	0.0360	0.108	-0.2455	0.0363	0.0379	0.000
β_3	-0.0450	0.0751	0.0712	0.879	-0.1081	0.0725	0.0678	0.629
σ^2	-0.3441	0.0582	NA	NA	-0.4844	0.0577	NA	NA
ρ	-0.0686	0.0163	NA	NA	-0.1101	0.0196	NA	NA

Table 12.2. Results of four parameter missing analyses under MNAR with missing rates in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(time$_{(3-1)}$) = β_2(group$_{(2-1)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: AR(1)

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0362	0.0580	0.0567	0.895	0.0915	0.0581	0.0557	0.618
β_1	-0.2108	0.0405	0.0393	0.000	-0.4173	0.0496	0.0421	0.000
β_2	-0.2155	0.0480	0.0463	0.007	-0.4390	0.0606	0.0474	0.000
β_3	-0.0724	0.0725	0.0676	0.780	-0.1842	0.0722	0.0613	0.181
σ^2	-0.3206	0.0560	NA	NA	-0.4925	0.0601	NA	NA
ρ	-0.1181	0.0213	NA	NA	-0.2150	0.0313	NA	NA
EM								
β_0	0.0235	0.0575	0.0573	0.929	0.0600	0.0561	0.0564	0.809
β_1	-0.1117	0.0366	0.0343	0.117	-0.2485	0.0378	0.0345	0.000
β_2	-0.1502	0.0441	0.0442	0.080	-0.3236	0.0479	0.0440	0.000
β_3	-0.0470	0.0726	0.0708	0.892	-0.1211	0.0657	0.0669	0.572
σ^2	-0.2802	0.0581	NA	NA	-0.4388	0.0583	NA	NA
ρ	-0.0336	0.0166	NA	NA	-0.0629	0.0191	NA	NA
LMM								
β_0	0.0234	0.0576	0.0548	0.920	0.0618	0.0564	0.0530	0.759
β_1	-0.1112	0.0356	0.0348	0.116	-0.2379	0.0357	0.0361	0.000
β_2	-0.1601	0.0441	0.0441	0.046	-0.3377	0.0461	0.0449	0.000
β_3	-0.0496	0.0730	0.0714	0.886	-0.1229	0.0680	0.0694	0.588
σ^2	-0.2581	0.0583	NA	NA	-0.3731	0.0609	NA	NA
ρ	-0.0550	0.0350	NA	NA	-0.0751	0.0364	NA	NA
GEE								
β_0	0.0234	0.0575	0.0573	0.935	0.0632	0.0563	0.0568	0.799
β_1	-0.1149	0.0357	0.0352	0.095	-0.2500	0.0358	0.0365	0.000
β_2	-0.1660	0.0442	0.0453	0.038	-0.3535	0.0463	0.0464	0.000
β_3	-0.0521	0.0727	0.0710	0.880	-0.1310	0.0676	0.0685	0.518
σ^2	-0.3104	0.0547	NA	NA	-0.4423	0.0555	NA	NA
ρ	-0.0756	0.0169	NA	NA	-0.1157	0.0199	NA	NA
PROPOSED								
β_0	0.0238	0.0577	0.0536	0.902	0.0615	0.0565	0.0513	0.747
β_1	-0.1071	0.0356	0.0355	0.144	-0.2340	0.0360	0.0372	0.000
β_2	-0.1495	0.0437	0.0454	0.091	-0.3213	0.0459	0.0467	0.000
β_3	-0.0477	0.0733	0.0690	0.873	-0.1242	0.0680	0.0660	0.538
σ^2	-0.3215	0.0544	NA	NA	-0.4577	0.0548	NA	NA
ρ	-0.0567	0.0147	NA	NA	-0.0909	0.0178	NA	NA

Table 12.3. Results of four parameter missing analyses under MNAR with missing rates in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(time$_{(3\text{-}1)}$) = β_2(group$_{(2\text{-}1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: Exchangeable

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0421	0.0530	0.0526	0.874	0.1011	0.0548	0.0519	0.494
β_1	-0.2114	0.0558	0.0544	0.029	-0.4172	0.0616	0.0535	0.000
β_2	-0.2158	0.0576	0.0544	0.030	-0.4405	0.0643	0.0534	0.000
β_3	-0.0806	0.0564	0.0542	0.683	-0.2028	0.0594	0.0500	0.029
σ^2	-0.3164	0.0469	NA	NA	-0.4888	0.0505	NA	NA
ρ	-0.0462	0.0200	NA	NA	-0.0798	0.0201	NA	NA
EM								
β_0	0.0392	0.0526	0.0526	0.886	0.0877	0.0524	0.0519	0.609
β_1	-0.2052	0.0562	0.0539	0.032	-0.4091	0.0558	0.0526	0.000
β_2	-0.2047	0.0557	0.0539	0.036	-0.4080	0.0562	0.0525	0.000
β_3	-0.0748	0.0548	0.0549	0.734	-0.1760	0.0508	0.0514	0.067
σ^2	-0.3173	0.0460	NA	NA	-0.4831	0.0438	NA	NA
ρ	-0.0292	0.0224	NA	NA	-0.0460	0.0236	NA	NA
LMM								
β_0	0.0405	0.0531	0.0498	0.858	0.0955	0.0536	0.0479	0.468
β_1	-0.2047	0.0549	0.0542	0.033	-0.4132	0.0537	0.0542	0.000
β_2	-0.2040	0.0556	0.0542	0.039	-0.4123	0.0544	0.0542	0.000
β_3	-0.0775	0.0561	0.0561	0.725	-0.1915	0.0537	0.0546	0.054
σ^2	-0.3061	0.0455	NA	NA	-0.4497	0.0441	NA	NA
ρ	-0.4384	0.0163	NA	NA	-0.4461	0.0220	NA	NA
GEE								
β_0	0.0406	0.0531	0.0529	0.881	0.0956	0.0536	0.0527	0.560
β_1	-0.2049	0.0549	0.0549	0.034	-0.4137	0.0537	0.0550	0.000
β_2	-0.2042	0.0556	0.0550	0.043	-0.4128	0.0544	0.0549	0.000
β_3	-0.0776	0.0561	0.0560	0.723	-0.1917	0.0537	0.0542	0.052
σ^2	-0.3092	0.0453	NA	NA	-0.4529	0.0439	NA	NA
ρ	-0.0308	0.0214	NA	NA	-0.0497	0.0224	NA	NA
PROPOSED								
β_0	0.0406	0.0531	0.0497	0.855	0.0957	0.0536	0.0478	0.463
β_1	-0.2051	0.0549	0.0543	0.033	-0.4140	0.0537	0.0544	0.000
β_2	-0.2043	0.0556	0.0543	0.039	-0.4131	0.0544	0.0544	0.000
β_3	-0.0777	0.0560	0.0559	0.724	-0.1919	0.0537	0.0543	0.051
σ^2	-0.3087	0.0452	NA	NA	-0.4520	0.0438	NA	NA
ρ	-0.0337	0.0221	NA	NA	-0.0525	0.0244	NA	NA

Table 12.4. Results of four parameter missing analyses under MNAR with missing rates in (group1, group2)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(time$_{(3\text{-}1)}$) = β_2(group$_{(2\text{-}1)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: AR(1)

Missing rate	(5%, 10%)				(10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
MI								
β_0	0.0389	0.0531	0.0520	0.872	0.1025	0.0537	0.0513	0.472
β_1	-0.2089	0.0549	0.0544	0.027	-0.4194	0.0618	0.0535	0.000
β_2	-0.2164	0.0630	0.0584	0.050	-0.4420	0.0679	0.0564	0.000
β_3	-0.0770	0.0544	0.0524	0.677	-0.2003	0.0609	0.0486	0.039
σ^2	-0.3178	0.0488	NA	NA	-0.4923	0.0501	NA	NA
ρ	-0.0521	0.0218	NA	NA	-0.0868	0.0212	NA	NA

Table 12.4. Continue

EM								
β_0	0.0367	0.0532	0.0520	0.876	0.0891	0.0511	0.0513	0.575
β_1	-0.2012	0.0550	0.0539	0.042	-0.4083	0.0572	0.0525	0.000
β_2	-0.2104	0.0617	0.0584	0.060	-0.4194	0.0585	0.0564	0.000
β_3	-0.0727	0.0532	0.0529	0.710	-0.1735	0.0498	0.0497	0.064
σ^2	-0.3189	0.0472	NA	NA	-0.4852	0.0445	NA	NA
ρ	-0.0346	0.0242	NA	NA	-0.0541	0.0258	NA	NA
LMM								
β_0	0.0361	0.0530	0.0492	0.862	0.0971	0.0521	0.0473	0.462
β_1	-0.1992	0.0540	0.0542	0.044	-0.4127	0.0550	0.0543	0.000
β_2	-0.2129	0.0608	0.0579	0.049	-0.4315	0.0571	0.0571	0.000
β_3	-0.0763	0.0540	0.0543	0.709	-0.1940	0.0526	0.0530	0.044
σ^2	-0.3070	0.0472	NA	NA	-0.4518	0.0444	NA	NA
ρ	-0.0442	0.0315	NA	NA	-0.0678	0.0334	NA	NA
GEE								
β_0	0.0360	0.0530	0.0523	0.886	0.0972	0.0521	0.0521	0.535
β_1	-0.1995	0.0540	0.0549	0.045	-0.4136	0.0550	0.0549	0.000
β_2	-0.2133	0.0608	0.0594	0.055	-0.4323	0.0572	0.0587	0.000
β_3	-0.0764	0.0540	0.0540	0.706	-0.1944	0.0526	0.0525	0.042
σ^2	-0.3109	0.0469	NA	NA	-0.4558	0.0442	NA	NA
ρ	-0.0380	0.0230	NA	NA	-0.0637	0.0239	NA	NA
PROPOSED								
β_0	0.0377	0.0530	0.0492	0.857	0.0981	0.0521	0.0474	0.454
β_1	-0.2004	0.0541	0.0542	0.042	-0.4127	0.0550	0.0542	0.000
β_2	-0.2111	0.0608	0.0585	0.054	-0.4266	0.0572	0.0579	0.000
β_3	-0.0748	0.0541	0.0540	0.711	-0.1916	0.0527	0.0527	0.049
σ^2	-0.3073	0.0472	NA	NA	-0.4521	0.0445	NA	NA
ρ	-0.0289	0.0243	NA	NA	-0.0464	0.0270	NA	NA

Table 13.1. Results of five parameter missing analyses under MNAR with missing rates in (group11, group12, group21, group22)

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(group$_{(31\text{-}11)}$) = β_2(group$_{(22\text{-}12)}$) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	(5%, 5%, 10%, 25%)				(5%, 10%, 10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
LMM								
β_0	0.0477	0.4428	0.4484	0.950	0.0716	0.4524	0.4461	0.941
β_1	-0.1463	0.0358	0.0361	0.011	-0.1644	0.0365	0.0364	0.003
β_2	-0.0622	0.0791	0.0788	0.870	-0.0480	0.0774	0.0784	0.905
β_3	-0.0324	0.0789	0.0788	0.924	-0.0493	0.0738	0.0784	0.926
β_4	0.0000	0.0096	0.0098	0.955	-0.0005	0.0099	0.0098	0.946
σ^2	-0.1842	0.0700	NA	NA	-0.2031	0.0706	NA	NA
ρ	-0.1479	0.0198	NA	NA	-0.1519	0.0217	NA	NA
Random Effect Pattern Mixture Model								
β_0	-0.0335	0.4369	0.4330	0.945	-0.0333	0.4538	0.4294	0.933
β_1	-0.0819	0.0357	0.0359	0.391	-0.0919	0.0363	0.0361	0.291
β_2	-0.2331	0.0772	0.0764	0.136	-0.1842	0.0751	0.0756	0.334
β_3	-0.1263	0.0772	0.0762	0.614	-0.1858	0.0744	0.0756	0.297
β_4	0.0001	0.0095	0.0095	0.948	-0.0003	0.0100	0.0094	0.938
σ^2	-0.4320	0.0560	NA	NA	-0.4733	0.0553	NA	NA
ρ	-0.1949	0.0287	NA	NA	-0.2076	0.0318	NA	NA
PROPOSED								
β_0	0.0501	0.4419	0.4361	0.942	0.0741	0.4513	0.4326	0.930
β_1	-0.1533	0.0359	0.0373	0.007	-0.1734	0.0367	0.0377	0.002
β_2	-0.0653	0.0789	0.0766	0.848	-0.0508	0.0771	0.0760	0.891
β_3	-0.0341	0.0787	0.0766	0.917	-0.0521	0.0735	0.0760	0.906
β_4	0.0000	0.0096	0.0096	0.951	-0.0005	0.0099	0.0095	0.940
σ^2	-0.2497	0.0631	NA	NA	-0.2743	0.0633	NA	NA
ρ	-0.0598	0.0193	NA	NA	-0.0687	0.0201	NA	NA

Table 14.1. Results of five parameter proposed approach with spline under MCAR assuming the same missing rate in each group

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(group$_{(31\text{-}11)}$) = β_2(group$_{(22\text{-}12)}$) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: Exchangeable

Missing rate	(5%, 5%, 10%, 25%)				(5%, 10%, 10%, 25%)			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
LMM								
β_0	0.0956	0.3734	0.3740	0.946	0.0988	0.3802	0.3724	0.941
β_1	-0.2731	0.0559	0.0559	0.001	-0.3049	0.0556	0.0559	0.000
β_2	-0.1232	0.0660	0.0656	0.533	-0.0937	0.0634	0.0653	0.710
β_3	-0.0658	0.0648	0.0656	0.823	-0.0958	0.0664	0.0653	0.684
β_4	0.0000	0.0082	0.0082	0.951	-0.0001	0.0082	0.0082	0.951
σ^2	-0.2495	0.0577	NA	NA	-0.2706	0.0583	NA	NA
ρ	-0.1574	0.0210	NA	NA	-0.1583	0.0201	NA	NA
Random Effect Pattern Mixture Model								
β_0	0.0629	0.3820	0.3853	0.954	0.0595	0.3902	0.3852	0.945
β_1	-0.2226	0.0573	0.0570	0.025	-0.2484	0.0572	0.0572	0.009
β_2	-0.1668	0.0668	0.0678	0.316	-0.1284	0.0645	0.0677	0.518
β_3	-0.0921	0.0661	0.0677	0.710	-0.1314	0.0673	0.0677	0.490
β_4	0.0001	0.0084	0.0084	0.955	0.0000	0.0084	0.0084	0.948
σ^2	-0.2666	0.0564	NA	NA	-0.2895	0.0582	NA	NA
ρ	-0.1585	0.0207	NA	NA	-0.1597	0.0200	NA	NA
PROPOSED								
β_0	0.0958	0.3734	0.3735	0.945	0.0987	0.3794	0.3717	0.941
β_1	-0.2733	0.0559	0.0560	0.001	-0.3051	0.0556	0.0560	0.000
β_2	-0.1232	0.0659	0.0655	0.530	-0.0938	0.0634	0.0652	0.709
β_3	-0.0658	0.0648	0.0655	0.823	-0.0958	0.0663	0.0652	0.684
β_4	0.0000	0.0082	0.0082	0.951	-0.0001	0.0082	0.0081	0.951
σ^2	-0.2507	0.0571	NA	NA	-0.2720	0.0575	NA	NA
ρ	-0.0341	0.0316	NA	NA	-0.0371	0.0315	NA	NA

Table 14.1. Estimates from the proposed model incorporating B-spline function with each of 3,5 or 7 knotsunder MCARassuming the same missing rate in each group

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(group$_{(31\text{-}11)}$) = β_3(group$_{(22\text{-}12)}$) =β_3(continuous) = 0, σ^2 = 2, ρ = 0.2, Correlation structure: Exchangeable

Missing rate	10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
PROPOSED								
β_0	0.0417	0.0680	0.0667	0.895	0.0417	0.0729	0.0713	0.911
β_1	-0.0003	0.0432	0.0428	0.946	-0.0022	0.0476	0.0490	0.954
β_2	-0.0046	0.0871	0.0728	0.900	-0.0021	0.0863	0.0772	0.919
β_3	-0.5284	0.1052	0.0853	0.000	-0.5815	0.1130	0.0916	0.000
β_4	0.2799	0.0294	0.0236	0.000	0.3095	0.0336	0.0263	0.000
σ^2	-0.3542	0.0644	NA	NA	-0.3427	0.0705	NA	NA
ρ	-0.1740	0.0264	NA	NA	-0.1737	0.0318	NA	NA
PROPOSED_splines (3 knots)								
β_0	-2.8443	0.5174	0.5505	0.003	-2.8138	0.5471	0.5789	0.004
β_1	0.0009	0.0387	0.0337	0.909	-0.0001	0.0435	0.0379	0.915
β_2	-0.0045	0.0645	0.0719	0.967	0.0001	0.0676	0.0756	0.974
β_3	-0.0042	0.0633	0.0719	0.981	0.0018	0.0662	0.0756	0.974
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.5230	0.0583	NA	NA	-0.5206	0.0635	NA	NA
ρ	-0.0220	0.0199	NA	NA	-0.0016	0.0222	NA	NA

Table 14.1. Continue

PROPOSED_splines (5 knots)								
β_0	-3.1965	0.6969	0.7419	0.008	-3.1901	0.7580	0.7938	0.012
β_1	0.0009	0.0387	0.0337	0.910	-0.0001	0.0435	0.0379	0.916
β_2	-0.0043	0.0644	0.0719	0.967	0.0001	0.0674	0.0756	0.975
β_3	-0.0044	0.0634	0.0719	0.981	0.0020	0.0662	0.0756	0.974
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.5263	0.0586	NA	NA	-0.5219	0.0638	NA	NA
ρ	-0.0233	0.0200	NA	NA	-0.0021	0.0223	NA	NA
PROPOSED_splines (7 knots)								
β_0	-3.2227	0.7584	0.8368	0.017	-3.2177	0.8588	0.9222	0.026
β_1	0.0009	0.0387	0.0337	0.910	-0.0001	0.0435	0.0379	0.915
β_2	-0.0045	0.0645	0.0720	0.969	0.0001	0.0675	0.0757	0.975
β_3	-0.0043	0.0634	0.0720	0.980	0.0020	0.0661	0.0757	0.974
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.5267	0.0586	NA	NA	-0.5223	0.0637	NA	NA
ρ	-0.0237	0.0201	NA	NA	-0.0025	0.0223	NA	NA

Table 14.2. Estimates from the proposed model incorporating B-spline function with each of 3,5 or 7 knotsunder MARassuming the same missing rate in each group

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2\text{-}1)}$) = β_2(group$_{(31\text{-}11)}$) = β_3(group$_{(22\text{-}12)}$) =β_3(continuous) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
PROPOSED								
β_0	0.0254	0.1879	0.1866	0.907	0.0251	0.195	0.1933	0.906
β_1	-0.0301	0.0507	0.0408	0.834	-0.0720	0.0899	0.0461	0.550
β_2	-0.0006	0.0811	0.0767	0.926	0.0003	0.0826	0.0786	0.924
β_3	-0.2734	0.3107	0.0823	0.516	-0.3018	0.3465	0.0849	0.516
β_4	0.1431	0.1574	0.0162	0.509	0.1581	0.1737	0.0171	0.519
σ^2	-0.2104	0.2283	NA	NA	-0.2269	0.2348	NA	NA
ρ	-0.1282	0.1346	NA	NA	-0.1538	0.156	NA	NA
PROPOSED_splines (3 knots)								
β_0	-1.2845	1.517	0.5291	0.508	-1.3017	1.5117	0.5404	0.506
β_1	-0.0023	0.0371	0.0365	0.945	0.0041	0.0482	0.0413	0.906
β_2	-0.0003	0.0743	0.0755	0.953	0.0018	0.0738	0.0772	0.960
β_3	-0.0026	0.0758	0.0754	0.950	0.0014	0.0773	0.0772	0.955
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.3326	0.356	NA	NA	-0.3597	0.3721	NA	NA
ρ	-0.0643	0.0678	NA	NA	-0.077	0.0769	NA	NA
PROPOSED_splines (5 knots)								
β_0	-1.4636	1.7749	0.7539	0.507	-1.4855	1.7816	0.7762	0.504
β_1	-0.0023	0.0371	0.0365	0.946	0.0038	0.0481	0.0413	0.906
β_2	-0.0003	0.0743	0.0755	0.954	0.0019	0.0739	0.0772	0.958
β_3	-0.0024	0.0757	0.0755	0.953	0.0014	0.0773	0.0772	0.955
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.3334	0.3568	NA	NA	-0.3613	0.3738	NA	NA
ρ	-0.0647	0.0682	NA	NA	-0.0784	0.0783	NA	NA
PROPOSED_splines (7 knots)								
β_0	-1.4769	1.828	0.8471	0.513	-1.4996	1.8307	0.8839	0.513
β_1	-0.0022	0.0371	0.0366	0.945	0.0039	0.0481	0.0413	0.903
β_2	-0.0003	0.0744	0.0756	0.953	0.0018	0.0739	0.0773	0.957
β_3	-0.0025	0.0757	0.0756	0.950	0.0016	0.0775	0.0773	0.957
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.3335	0.3569	NA	NA	-0.3614	0.374	NA	NA
ρ	-0.0649	0.0683	NA	NA	-0.0786	0.0785	NA	NA

Table 14.3. Estimates from the proposed model incorporating B-spline function with each of 3,5 or 7 knotsunder MNARassuming the same missing rate in each group

Gold standard: β_0(Baseline) = 1, β_1(time$_{(2-1)}$) = β_2(group$_{(31-11)}$) = β_3(group$_{(22-12)}$) =β_3(continuous) = 0, σ^2 = 2, ρ = 0.7, Correlation structure: Exchangeable

Missing rate	10%				25%			
Parameter	Bias	Std.Dev	SE	CP	Bias	Std.Dev	SE	CP
PROPOSED								
β_0	-0.0076	0.2291	0.2244	0.943	-0.0034	0.2280	0.2160	0.940
β_1	-0.1522	0.0357	0.0371	0.017	-0.3582	0.0379	0.0416	0.000
β_2	-0.0004	0.0754	0.0759	0.946	-0.0002	0.0764	0.0731	0.941
β_3	0.0010	0.0766	0.0759	0.944	0.0000	0.0776	0.0731	0.935
β_4	0.0002	0.0049	0.0048	0.946	0.0000	0.0049	0.0046	0.931
σ^2	-0.2722	0.0638	NA	NA	-0.3954	0.0617	NA	NA
ρ	-0.0657	0.0193	NA	NA	-0.1274	0.0235	NA	NA
PROPOSED_splines (3 knots)								
β_0	0.0045	0.5222	0.5160	0.947	0.0220	0.5044	0.4915	0.942
β_1	-0.1521	0.0357	0.0371	0.017	-0.3579	0.0378	0.0416	0.000
β_2	-0.0004	0.0753	0.0760	0.948	-0.0002	0.0765	0.0731	0.943
β_3	0.0009	0.0767	0.0760	0.945	0.0000	0.0776	0.0731	0.933
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.2723	0.0639	NA	NA	-0.3955	0.0617	NA	NA
ρ	-0.0657	0.0194	NA	NA	-0.1275	0.0235	NA	NA
PROPOSED_splines (5 knots)								
β_0	0.0245	0.7819	0.7599	0.944	0.0245	0.7592	0.7252	0.936
β_1	-0.1520	0.0357	0.0371	0.017	-0.3576	0.0378	0.0416	0.000
β_2	-0.0003	0.0754	0.0760	0.947	-0.0001	0.0765	0.0732	0.942
β_3	0.0007	0.0766	0.0760	0.949	-0.0002	0.0776	0.0732	0.934
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.2725	0.0639	NA	NA	-0.3957	0.0618	NA	NA
ρ	-0.0657	0.0194	NA	NA	-0.1275	0.0235	NA	NA
PROPOSED_splines (7 knots)								
β_0	0.0140	0.8886	0.8546	0.950	0.0248	0.8676	0.8165	0.931
β_1	-0.1519	0.0357	0.0371	0.017	-0.3573	0.0378	0.0416	0.000
β_2	-0.0002	0.0754	0.0761	0.948	-0.0002	0.0767	0.0733	0.944
β_3	0.0006	0.0768	0.0761	0.946	-0.0001	0.0777	0.0733	0.934
β_4	NA	NA	NA	NA	NA	NA	NA	NA
σ^2	-0.2725	0.0639	NA	NA	-0.3958	0.0619	NA	NA
ρ	-0.0657	0.0195	NA	NA	-0.1275	0.0235	NA	NA