

Numerical Comparison of Thermal Stratification due Natural Convection in Densified LO_x and LN₂ Tanks

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Abstract: The growth of natural circulation and thermal stratification inside LO_x, LN₂ storage tanks due to heat transfer from surroundings is investigated using a non-equilibrium, two-domain, mathematical model. The two-dimensional model is considered for the liquid domain in two cases and a lumped, thermodynamic model is utilized for vapor domain. The vapor is assumed to behave like an ideal gas. The developed mathematical model for liquid domain consists of conservation of mass, momentum, energy equations with Boussinesq approximation. The coupling of two domains is carried out using energy balance at the interface. An implicit, finite-volume technique with a uniform mesh for liquid domain is used to predict the velocity and temperature fields.

Key words: Cryogenic, LO_x, LN₂, thermal stratification, natural convection

INTRODUCTION

The main objective of this article is to study the transient natural convection inside LN₂ and LO_x cryogenic tanks due to heat transfer from surroundings. In aerospace industry like Single-Stage-To-Orbit (SSTO) or Reusable Launch Vehicles (RLV) the volume/weight propellant storage tanks is of vital importance, due to its size in these vehicles. As a result, much effort has been carried out to increase the density of such propellant as much as possible.

In these cases the vapor and operating pressures are lowered to have a lighter main tank structure and reduces the structural reinforcement need which ultimately leads to smaller vehicle. As a result thermal protection systems requirement is lowered due to decrease in aerodynamic drag of the vehicle^[1].

A physical based transient mathematical model is developed to simulate initiation and growth of the natural circulation and thermal stratification which occurs inside a cryogenic tank. In this way different cryogenic liquid behavior is analyzed.

The developed mathematical model is based on conservation equations, relevant boundary and initial conditions in liquid domain and thermodynamic equations of ideal gas for vapor domain in this study. The energy balance is derived for the variable heat flux in the solid wall boundary as well as in the liquid-vapor interface which determines the interfacial liquid temperature as well as mass that evaporate from the liquid.

PHYSICAL MODEL

The basic physical model is developed for a vertical cryogenic propellant tank which is subjected to heat transfer from the surroundings as shown in Fig. 1 the tank is divided into the liquid and vapor domains at the ullage. A vent/relief and a pressurization valves are located at the top of the tank to control the tank pressure.

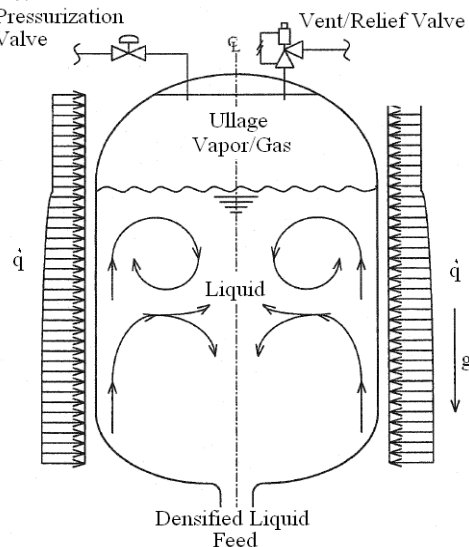


Fig. 1: Cryogenic tank subjected to external heating

The cryogenic tank is charged with sub-cooled liquid from bottom of the tank. Heat transfer is carried out to

the propellant from the tank walls due to poor insulation which creates density gradients that are orthogonal to the gravity vector. As a result, the warmer and consequently less dense liquid adjacent to the walls ascends past the cooler, heavier liquid and natural circulation phenomena occurs inside the tank.

The ullage pressure increases with rate of evaporation and directly affects the saturation temperature in vapor-liquid interface. Also, the heat flux from the surroundings is variable with time since temperature gradient between the surroundings and the fluid changes as a result fluid temperature increases. The geometry which is used in the present study is illustrated in Fig. 2 this rectangular section could represent a cross-section of a cubical or a cylindrical tank, where the three-dimensional effects are neglected. The simulation is carried out for a micro-scale cryogenic tank. The dimensions in this simulation are 75x100 mm, with initial liquid height of 75 mm. Only laminar flow is considered in the present study. Therefore, an effort is made to keep the Rayleigh numbers below the transition limit (between 10^7 - 10^{10}) to turbulent flow^[2,3,4]. Where these transition limit is highly affected by enclosure/tank configuration, working fluid and operating condition.

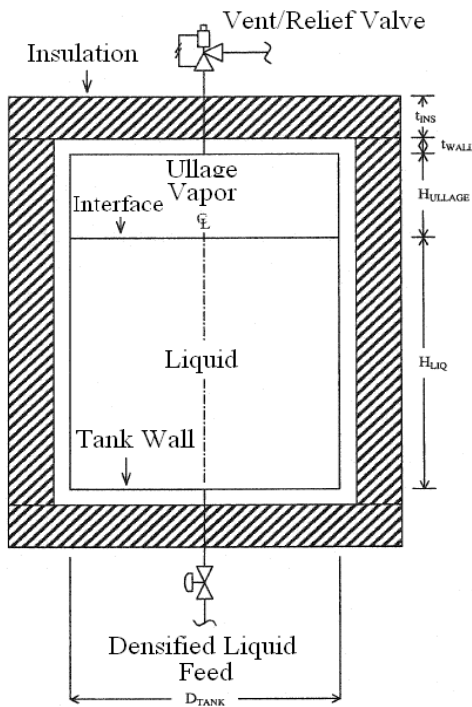


Fig. 2: Model tank applied in present study

The tank wall is chosen as lithium-aluminum alloy having a 3 mm thick. The properties of the alloy have been approximated using a weighted average of the properties of lithium and aluminum. The tank is insulated by a 25 mm thick material with properties similar to a calcium-silicate. Finally, the model of cryogenic tank is assumed to be outdoors at an average daily temperature of 30°C with no wind and at relatively low humidity. The vent/relief valve is set at 1 bar.

MATHEMATICAL MODEL

Simplifications of mathematical model can be attained by considering the symmetric nature of the physical problem about the centerline, vertical plane of the tank is depicted in Fig. 3. The following assumptions have been made in developing the mathematical model to simulate the onset and growth of the natural circulation and thermal stratification inside cryogenic propellant storage tank.

LIQUID DOMAIN

The fluid is treated as incompressible except in the buoyancy term in momentum equation where the density is taken as a function of temperature. The flow is laminar, viscous and unsteady. No liquid flow into or out of the tank occurs. The natural convection phenomenon can be approximated by using a two

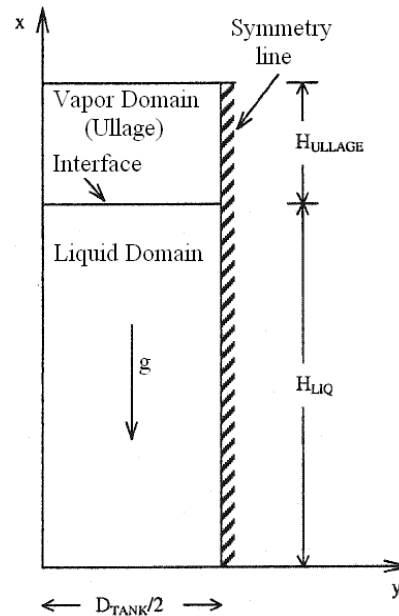


Fig. 3: Model cryogenic tank in cartesian coordinate

dimensional model in Cartesian coordinates^[6]. Boussinesq approximation is utilized. The governing, coupled, non-linear, partial differential equations using relevant assumptions,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial (P - P_0)}{\partial x} + g\beta(T - T_0) + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = -\frac{1}{\rho} \frac{k}{c_p} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

VAPOR DOMAIN (ULLAGE)

The vapor is modeled using a lumped or single node, thermodynamic approach. The vent/relief valve is fast response to maintain the set pressure. Heat transfer is in the form of convection to and from the liquid with variable heat flux from the surroundings through the tank walls. The vapor is initially superheated at a prescribed value. A unit depth scale is used as necessary.

The governing equations for the vapor domain considering a lumped, thermodynamic approach are as follows.

$$\frac{dm_{ULLAGE}}{dt} = \Sigma \dot{m}_{in} - \Sigma \dot{m}_e$$

The single mass source, evaporation from the liquid domain and a single mass sink discharge from the vent/relief valve is considered. In uniform state and flow, the thermodynamic energy equation becomes;

$$\begin{aligned} \frac{dE_{ULLAGE}}{dt} &= \dot{Q}_{ULLAGE} - \dot{W}_{ULLAGE} \\ &+ \Sigma \dot{m}_{in} \left(h_{in} + \frac{\bar{V}_{in}^2}{2} + gZ_{in} \right) \\ &- \Sigma \dot{m}_e \left(h_e + \frac{\bar{V}_e^2}{2} + gZ_e \right) \end{aligned} \tag{6}$$

BOUNDARY AND INITIAL CONDITIONS

Appropriate boundary and the initial conditions for liquid and vapor domain, needed for mathematical formulation are listed below.

Vertical velocity components at the interface and bottom and vertical wall of tank are zero due to no slip condition. Liquid at the bottom of tank is assumed at sub-cooled temperature. The fluid is assumed initially is at rest. Hydrostatic pressure exists in the liquid initially. The initial ullage pressure is as specified and temperature is initialized to a super heated value.

INTERFACE ENERGY BALANCE

The liquid-vapor interface is assumed planner and horizontal while evaporation occurs. The interfacial energy balance through convective heat transfer occurs between liquid and ullage vapor. The empirical equation for the vapor convection heat transfer coefficient as a function of Rayleigh number is used^[7]. Figure. 4 shows the nonlinear heat transfer which is high lighted in this work, the energy balance for liquid-vapor interface becomes,

$$\dot{Q}_{CONV} - \dot{Q}_{COND_L} - \dot{m}_{evap} h_{fg} = 0 \tag{7}$$

Liquid evaporates when the interface temperature is at the saturation temperature corresponding to the

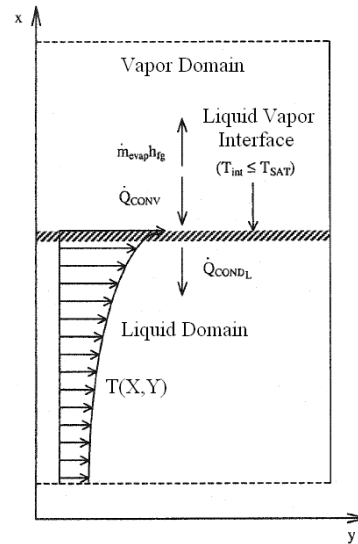


Fig. 4: Schematic of liquid vapor interfacial energy balance

interface pressure, where energy exceeds the saturated liquid state. The energy balance above is the result of convective and conduction heat transfer between vapor and liquid.

VARIABLE HEAT FLUX BOUNDARY CONDITIONS

The cryogenic storage tank is assumed to be imperfectly insulated such that heat is transferred from the surroundings to the fluid. The energy balance has following terms.

The convection from the ambient fluid i.e., air in this study, to the outer surface of the insulation also the conduction through insulation and tank wall.

It is assumed that the inner wall surface temperature at any point is the same as the fluid in contact. In this analysis, the heat capacities of the insulation and the tank wall are neglected. More specifically, no energy is absorbed by the insulation and the tank wall as heat is transferred,

$$\dot{Q}_{CONV} = \dot{Q}_{COND} = \dot{Q} \tag{8}$$

$$h_{CONV_{\infty}} A_{CONV} (T_{\infty} - T_{INS_0}) = k_{INS} \frac{A_{COND}}{t_{INS}}$$

$$(T_{INS_0} - T_{WALL_0}) = k_{WALL} \frac{A_{COND}}{t_{WALL}} \tag{9}$$

$$(T_{WALL_0} - T_{FLUID}) = \dot{Q}$$

This variable heat flux condition is applied along the vertical walls for the liquid domain and in vapor domain which is not wetted by fluid.

RESULT AND DISCUSSION

The LN₂ and LO_x are investigated in this study as liquefied nitrogen and oxygen fluid and heat transfer criteria and behavior are compared. As observed in (Fig. 5a-f) in two case, the heated liquid (near tank

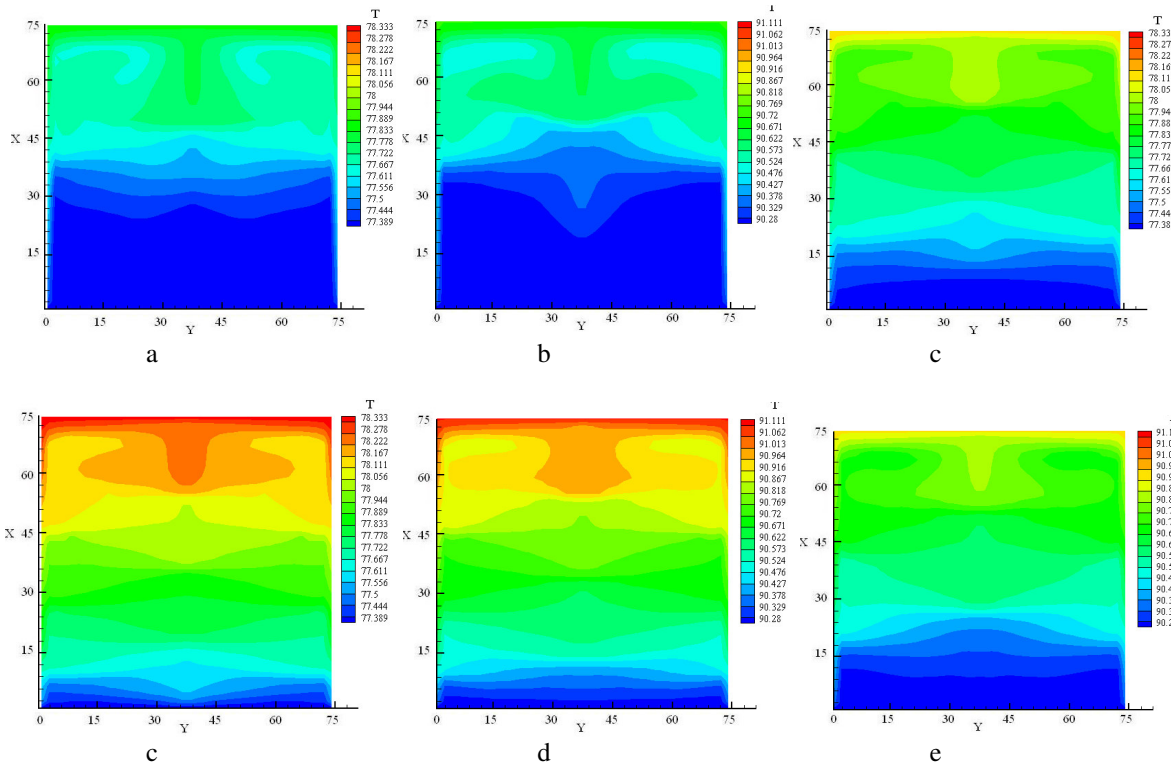


Fig. 5: Temperature contours (K) at t = 30, 60, 90 sec (LO_x, LN₂, Pr = 2.2 Ra = 1.97×10⁹), a: Temperature contour at t = 30 s (LO_x), b: Temperature contour at t = 30 s (LN₂), c: Temperature contour at t = 60 s (LO_x), d: Temperature contour at t = 60 s (LN₂), e: Temperature contour at t = 90 s (LO_x), f: Temperature contour at t = 90 s (LN₂)

walls) becomes less dense and rises above the adjacent cooler liquid eventually leading to a natural circulation within the tank. After reaching the top surface it turns towards the center. Due to symmetry of the tank, it then turns towards the bottom. But as the fluid descends, it starts losing momentum due to viscous and buoyancy effects against the direction of fluid motion and turn again towards the heated wall. As the fluid approaches the heated wall, it again bends downward before changing its direction back to the center of tank. This downward movement near the wall suggests that the temperature of circulating fluid is higher than that of layer adjacent to the boundary layer along the wall. This is possible because in those cases, the wall temperature increases with its height^[5] and hence the circulating fluid is hottest when it reaches the top of tank. The natural circulation that develops is directly related to the temperature field of the fluid. More

specifically, thermal stratification or uniform temperature in horizontal planes could develop in the core of the fluid (liquid).

In addition, liquid that evaporates at the liquid-vapor interface increases the pressure in the tank ullage as a result of mass and energy transfer. This pressure increase in the vapor ultimately affects the liquid domain, including the operating pressure and the saturation temperature of the interfacial liquid.

The velocity field and temperature contours (Fig. 6a-f) are generally similar to those of LN₂ after 30, 60 and 90 seconds, hot spots are exhibited at the bottom corner of the tank as in LN₂. The similarity in the fluid behavior is a consequence of similar thermo-physical properties in the two fluids. The main differences are saturation density and temperature. At 1 bar tank pressure these are 808.9 kg m⁻³ and 77.4°K for nitrogen, 1140.5 kg m⁻³ and 90.2°K, for oxygen.

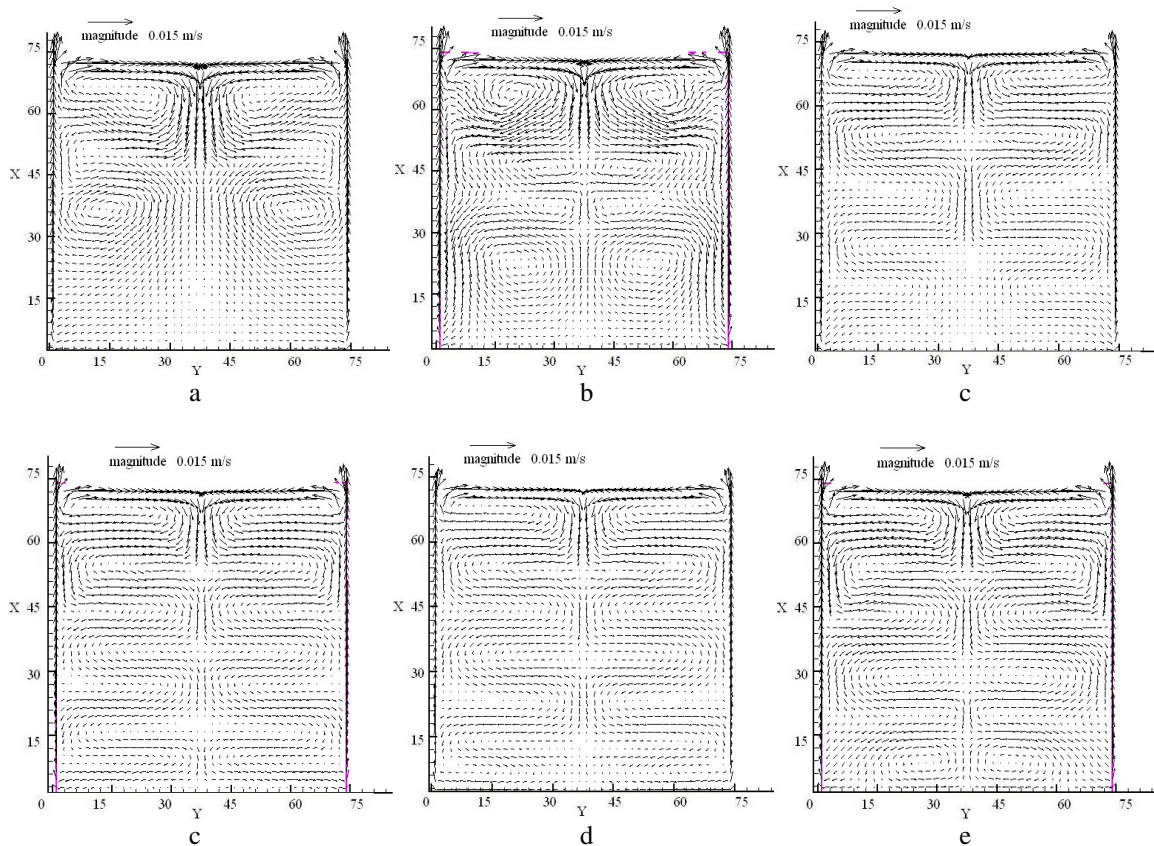


Fig. 6: Velocity fields for 30, 60, 90 s (LO_x, LN₂, Pr = 2.2 Ra = 1.97×10⁹), a: Velocity field at t = 30 s (LO_x), b: Velocity field at t = 30 s (LN₂), c: Velocity field at t = 60 s (LO_x), d: Velocity field at t = 60 s (LN₂), e: Velocity field at t = 90 s (LO_x), f: Velocity field at t = 90 s (LN₂)

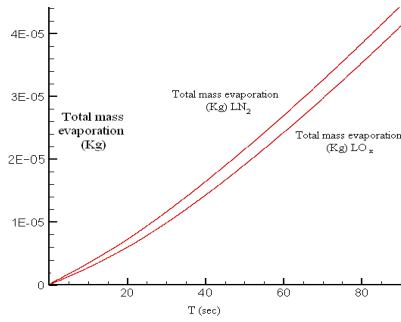


Fig. 7: Total mass evaporation in t = 90 s (LO_x , LN_2 , $Pr = 2.2$ $Ra = 1.97 \times 10^9$)

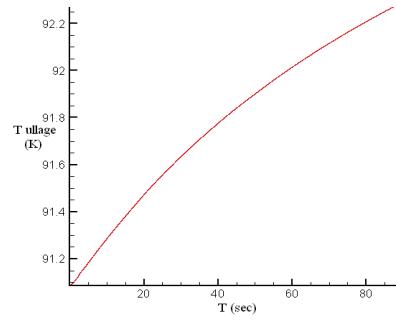


Fig. 9: Ullage temperature histogram in t = 90 s (LO_x , $Pr = 2.2$ $Ra = 1.97 \times 10^9$)

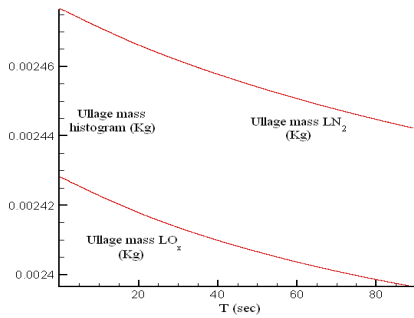


Fig. 8: Ullage mass histogram in t = 90 s (LO_x , LN_2 , $Pr = 2.2$ $Ra = 1.97 \times 10^9$)

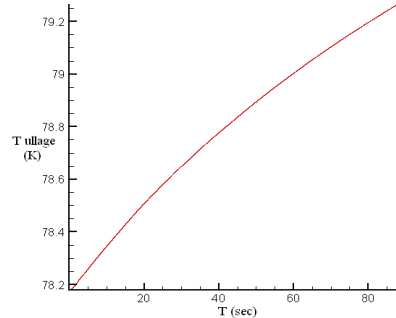


Fig. 10: Temperature histogram in t = 90 s (LN_2 , $Pr = 2.2$ $Ra = 1.97 \times 10^9$)

The lower operating temperatures for the nitrogen case results in a greater wall heat flux. Also mass evaporation for LN_2 exceeds that of LO_x (Fig. 7) due to the lower interfacial saturation temperature and enthalpy of evaporation for nitrogen. The enthalpy of evaporation for nitrogen is 198.8 and 212.4 $Kj\ kg^{-1}$ for oxygen.

As in the oxygen case, cooler fluid at core circulating and proceeding upward along the vertical walls and near the interface. As discussed earlier, the wall heat flux increase and the mass of evaporation decrease (as a consequence of a lower amount of heat transferred from this cooler fluid for interfacial evaporation).

Wall heat flux and mass of evaporation for the nitrogen case may be caused by the higher ambient heat flux to the liquid and nitrogen having a greater heat capacity than oxygen (at the operating conditions). The fluid temperature does not fall to the same degree as in LO_x after losing heat to the interfacial liquid, which is maintained at 77.4°K, Finally, the greater mass of evaporation in the nitrogen case provides for a greater ullage mass than in case LO_x (Fig. 8). The ideal gas equation of state gives an opposite trend in the ullage vapor temperature as shown in (Fig. 9, 10).

CONCLUSION

A non-equilibrium, two-domain, mathematical model is developed to investigate the growth of the natural circulation and thermal stratification inside LO_x , LN_2 storage tanks due to heat transfer from the surroundings. a two-dimensional model is incorporated for the liquid domain (in two cryogenic liquid) while a lumped, thermodynamic model is utilized for the vapor domain. Thermal stratification is very much dependent of natural circulation For operating conditions in this study the fluid behavior is generally similar for oxygen and nitrogen.

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